Solution to Problem 1.31. (a) Let A be the event that a 0 is transmitted. Using the total probability theorem, the desired probability is

$$\mathbf{P}(A)(1-\epsilon_0) + \big(1-\mathbf{P}(A)\big)(1-\epsilon_1) = p(1-\epsilon_0) + (1-p)(1-\epsilon_1).$$

(b) By independence, the probability that the string 1011 is received correctly is

$$(1-\epsilon_0)(1-\epsilon_1)^3.$$

(c) In order for a 0 to be decoded correctly, the received string must be 000, 001, 010, or 100. Given that the string transmitted was 000, the probability of receiving 000 is $(1-\epsilon_0)^3$, and the probability of each of the strings 001, 010, and 100 is $\epsilon_0(1-\epsilon_0)^2$. Thus, the probability of correct decoding is

$$3\epsilon_0(1-\epsilon_0)^2 + (1-\epsilon_0)^3.$$

(d) When the symbol is 0, the probabilities of correct decoding with and without the scheme of part (c) are $3\epsilon_0(1-\epsilon_0)^2 + (1-\epsilon_0)^3$ and $1-\epsilon_0$, respectively. Thus, the probability is improved with the scheme of part (c) if

$$3\epsilon_0(1-\epsilon_0)^2 + (1-\epsilon_0)^3 > (1-\epsilon_0),$$

or

$$(1-\epsilon_0)(1+2\epsilon_0) > 1,$$

which is equivalent to $\epsilon_0 < 1/2$.

(e) Using Bayes' rule, we have

$$\mathbf{P}(0\,|\,101) = \frac{\mathbf{P}(0)\mathbf{P}(101\,|\,0)}{\mathbf{P}(0)\mathbf{P}(101\,|\,0) + \mathbf{P}(1)\mathbf{P}(101\,|\,1)}.$$

The probabilities needed in the above formula are

$$\mathbf{P}(0) = p, \quad \mathbf{P}(1) = 1-p, \quad \mathbf{P}(101\,|\,0) = \epsilon_0^2(1-\epsilon_0), \quad \mathbf{P}(101\,|\,1) = \epsilon_1(1-\epsilon_1)^2.$$

Solution to Problem 1.32. The answer to this problem is not unique and depends on the assumptions we make on the reproductive strategy of the king's parents.

Suppose that the king's parents had decided to have exactly two children and then stopped. There are four possible and equally likely outcomes, namely BB, GG, BG, and GB (B stands for "boy" and G stands for "girl"). Given that at least one child was a boy (the king), the outcome GG is eliminated and we are left with three equally likely outcomes (BB, BG, and GB). The probability that the sibling is male (the conditional probability of BB) is 1/3 .

Suppose on the other hand that the king's parents had decided to have children until they would have a male child. In that case, the king is the second child, and the sibling is female, with certainty.

Solution to Problem 1.33. Flip the coin twice. If the outcome is heads-tails, choose the opera. if the outcome is tails-heads, choose the movies. Otherwise, repeat the process, until a decision can be made. Let A_k be the event that a decision was made at the kth round. Conditional on the event A_k, the two choices are equally likely, and we have

$$\mathbf{P}(\text{opera}) = \sum_{k=1}^{\infty} \mathbf{P}(\text{opera} \,|\, A_k)\mathbf{P}(A_k) = \sum_{k=1}^{\infty} \frac{1}{2}\mathbf{P}(A_k) = \frac{1}{2}.$$

We have used here the property $\sum_{k=0}^{\infty} \mathbf{P}(A_k) = 1$, which is true as long as $\mathbf{P}(\text{heads}) > 0$ and $\mathbf{P}(\text{tails}) > 0$.

Solution to Problem 1.34. The system may be viewed as a series connection of three subsystems, denoted 1, 2, and 3 in Fig. 1.19 in the text. The probability that the entire system is operational is $p_1 p_2 p_3$, where p_i is the probability that subsystem i is operational. Using the formulas for the probability of success of a series or a parallel system given in Example 1.24, we have

$$p_1 = p, \qquad p_3 = 1 - (1-p)^2,$$

and

$$p_2 = 1 - (1-p)\big(1 - p\big(1 - (1-p)^3\big)\big).$$

Solution to Problem 1.35. Let A_i be the event that exactly i components are operational. The probability that the system is operational is the probability of the union $\cup_{i=k}^{n} A_i$, and since the A_i are disjoint, it is equal to

$$\sum_{i=k}^{n} \mathbf{P}(A_i) = \sum_{i=k}^{n} p(i),$$

where $p(i)$ are the binomial probabilities. Thus, the probability of an operational system is

$$\sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i}.$$

Solution to Problem 1.36. (a) Let A denote the event that the city experiences a black-out. Since the power plants fail independent of each other, we have

$$\mathbf{P}(A) = \prod_{i=1}^{n} p_i.$$

(b) There will be a black-out if either all n or any $n-1$ power plants fail. These two events are disjoint, so we can calculate the probability $\mathbf{P}(A)$ of a black-out by adding their probabilities:

$$\mathbf{P}(A) = \prod_{i=1}^{n} p_i + \sum_{i=1}^{n} \left((1-p_i) \prod_{j \neq i} p_j \right).$$

Here, $(1-p_i)\prod_{j\neq i} p_j$ is the probability that $n-1$ plants have failed and plant i is the one that has not failed.

Solution to Problem 1.37. The probability that k_1 voice users and k_2 data users simultaneously need to be connected is $p_1(k_1)p_2(k_2)$, where $p_1(k_1)$ and $p_2(k_2)$ are the corresponding binomial probabilities, given by

$$p_i(k_i) = \binom{n_i}{k_i} p_i^{k_i}(1-p_i)^{n_i-k_i}, \qquad i = 1, 2.$$

The probability that more users want to use the system than the system can accommodate is the sum of all products $p_1(k_1)p_2(k_2)$ as k_1 and k_2 range over all possible values whose total bit rate requirement $k_1r_1+k_2r_2$ exceeds the capacity c of the system. Thus, the desired probability is

$$\sum_{\{(k_1,k_2)\,|\,k_1r_1+k_2r_2>c,\;k_1\leq n_1,\;k_2\leq n_2\}} p_1(k_1)p_2(k_2).$$

Solution to Problem 1.38. We have

$$p_T = \mathbf{P}(\text{at least 6 out of the 8 remaining holes are won by Telis}),$$

$$p_W = \mathbf{P}(\text{at least 4 out of the 8 remaining holes are won by Wendy}).$$

Using the binomial formulas,

$$p_T = \sum_{k=6}^{8} \binom{8}{k} p^k(1-p)^{8-k}, \qquad p_W = \sum_{k=4}^{8} \binom{8}{k} (1-p)^k p^{8-k}.$$

The amount of money that Telis should get is $10 \cdot p_T/(p_T+p_W)$ dollars.

Solution to Problem 1.39. Let the event A be the event that the professor teaches her class, and let B be the event that the weather is bad. We have

$$\mathbf{P}(A) = \mathbf{P}(B)\mathbf{P}(A\,|\,B) + \mathbf{P}(B^c)\mathbf{P}(A\,|\,B^c),$$

and

$$\mathbf{P}(A\,|\,B) = \sum_{i=k}^{n} \binom{n}{i} p_b^i(1-p_b)^{n-i},$$

$$\mathbf{P}(A\,|\,B^c) = \sum_{i=k}^{n} \binom{n}{i} p_g^i(1-p_g)^{n-i}.$$

Therefore,

$$\mathbf{P}(A) = \mathbf{P}(B)\sum_{i=k}^{n} \binom{n}{i} p_b^i(1-p_b)^{n-i} + \big(1-\mathbf{P}(B)\big)\sum_{i=k}^{n} \binom{n}{i} p_g^i(1-p_g)^{n-i}.$$

not a king is 47/51. We continue similarly until the 12th card. The probability that the 12th card is not a king, given that none of the preceding 11 was a king, is 37/41. (There are $52-11=41$ cards left, and $48-11=37$ of them are not kings.) Finally, the conditional probability that the 13th card is a king is 4/40. The desired probability is

$$\frac{48 \cdot 47 \cdots 37 \cdot 4}{52 \cdot 51 \cdots 41 \cdot 40}.$$

Solution to Problem 1.53. Suppose we label the classes A, B, and C. The probability that Joe and Jane will both be in class A is the number of possible combinations for class A that involve both Joe and Jane, divided by the total number of combinations for class A. Therefore, this probability is

$$\frac{\binom{88}{28}}{\binom{90}{30}}.$$

Since there are three classes, the probability that Joe and Jane end up in the same class is

$$3 \cdot \frac{\binom{88}{28}}{\binom{90}{30}}.$$

A much simpler solution is as follows. We place Joe in one class. Regarding Jane, there are 89 possible "slots", and only 29 of them place her in the same class as Joe. Thus, the answer is 29/89, which turns out to agree with the answer obtained earlier.

Solution to Problem 1.54. (a) Since the cars are all distinct, there are 20! ways to line them up.

(b) To find the probability that the cars will be parked so that they alternate, we count the number of "favorable" outcomes, and divide by the total number of possible outcomes found in part (a). We count in the following manner. We first arrange the US cars in an ordered sequence (permutation). We can do this in 10! ways, since there are 10 distinct cars. Similarly, arrange the foreign cars in an ordered sequence, which can also be done in 10! ways. Finally, interleave the two sequences. This can be done in two different ways, since we can let the first car be either US-made or foreign. Thus, we have a total of $2 \cdot 10! \cdot 10!$ possibilities, and the desired probability is

$$\frac{2 \cdot 10! \cdot 10!}{20!}.$$

Note that we could have solved the second part of the problem by neglecting the fact that the cars are distinct. Suppose the foreign cars are indistinguishable, and also that the US cars are indistinguishable. Out of the 20 available spaces, we need to choose 10 spaces in which to place the US cars, and thus there are $\binom{20}{10}$ possible outcomes. Out of these outcomes, there are only two in which the cars alternate, depending on from a l-letter alphabet is equal to

$$l!\binom{l-1}{w-1}.$$

Solution to Problem 1.58. (a) The sample space consists of all ways of drawing 7 elements out of a 52-element set, so it contains $\binom{52}{7}$ possible outcomes. Let us count those outcomes that involve exactly 3 aces. We are free to select any 3 out of the 4 aces, and any 4 out of the 48 remaining cards, for a total of $\binom{4}{3}\binom{48}{4}$ choices. Thus,

$$\mathbf{P}(\text{7 cards include exactly 3 aces}) = \frac{\binom{4}{3}\binom{48}{4}}{\binom{52}{7}}.$$

(b) Proceeding similar to part (a), we obtain

$$\mathbf{P}(\text{7 cards include exactly 2 kings}) = \frac{\binom{4}{2}\binom{48}{5}}{\binom{52}{7}}.$$

(c) If A and B stand for the events in parts (a) and (b), respectively, we are looking for $\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B) - \mathbf{P}(A \cap B)$. The event $A \cap B$ (having exactly 3 aces and exactly 2 kings) can occur by choosing 3 out of the 4 available aces, 2 out of the 4 available kings, and 2 more cards out of the remaining 44. Thus, this event consists of $\binom{4}{3}\binom{4}{2}\binom{44}{2}$ distinct outcomes. Hence,

$$\mathbf{P}(\text{7 cards include 3 aces and/or 2 kings}) = \frac{\binom{4}{3}\binom{48}{4} + \binom{4}{2}\binom{48}{5} - \binom{4}{3}\binom{4}{2}\binom{44}{2}}{\binom{52}{7}}.$$

Solution to Problem 1.59. Clearly if $n > m$, or $n > k$, or $m - n > 100 - k$, the probability must be zero. If $n \leq m$, $n \leq k$, and $m - n \leq 100 - k$, then we can find the probability that the test drive found n of the 100 cars defective by counting the total number of size m subsets, and then the number of size m subsets that contain n lemons. Clearly, there are $\binom{100}{m}$ different subsets of size m. To count the number of size m subsets with n lemons, we first choose n lemons from the k available lemons, and then choose $m - n$ good cars from the $100 - k$ available good cars. Thus, the number of ways to choose a subset of size m from 100 cars, and get n lemons, is

$$\binom{k}{n}\binom{100-k}{m-n},$$

and the desired probability is

$$\frac{\binom{k}{n}\binom{100-k}{m-n}}{\binom{100}{m}}.$$

Solution to Problem 1.60. The size of the sample space is the number of different ways that 52 objects can be divided in 4 groups of 13, and is given by the multinomial formula

$$\frac{52!}{13!\,13!\,13!\,13!}.$$

There are 4! different ways of distributing the 4 aces to the 4 players, and there are

$$\frac{48!}{12!\,12!\,12!\,12!}$$

different ways of dividing the remaining 48 cards into 4 groups of 12. Thus, the desired probability is

$$\frac{4!\,\dfrac{48!}{12!\,12!\,12!\,12!}}{\dfrac{52!}{13!\,13!\,13!\,13!}}.$$

An alternative solution can be obtained by considering a different, but probabilistically equivalent method of dealing the cards. Each player has 13 slots, each one of which is to receive one card. Instead of shuffling the deck, we place the 4 aces at the top, and start dealing the cards one at a time, with each free slot being equally likely to receive the next card. For the event of interest to occur, the first ace can go anywhere; the second can go to any one of the 39 slots (out of the 51 available) that correspond to players that do not yet have an ace; the third can go to any one of the 26 slots (out of the 50 available) that correspond to the two players that do not yet have an ace; and finally, the fourth, can go to any one of the 13 slots (out of the 49 available) that correspond to the only player who does not yet have an ace. Thus, the desired probability is

$$\frac{39\cdot 26\cdot 13}{51\cdot 50\cdot 49}.$$

By simplifying our previous answer, it can be checked that it is the same as the one obtained here, thus corroborating the intuitive fact that the two different ways of dealing the cards are probabilistically equivalent.

Prob 8

A — the San Andreas fault
B — the Calaveras fault
$P(A) = 0.21$ $P(B) = 0.11$

a → at least one major earthquake along the San Andreas fault or the calaveras fault

$P(A)P(B^c) + P(B)P(A^c) + P(A)P(B)$

$(0.21)(.89) + (0.11)(0.79) + (0.21)(0.11)$

b. C — the Hayward fault

$P(C) = 0.27$

→ not get hit by an earthquake along the San Andreas fault nor the Calveras fault but you do get hit by at least one major earthquake along the Hayward fault

$P(A^c)P(B^c)P(C) = (1 - 0.21)(1 - 0.11)(0.27)$

$= 0.189$

```
clear all, close all;
%% load sequences in varible
%%
load sequence1.mat;
sequence1 = test_sequence;

p_1 = sum ( 1 == conv ( 1  /1 * [1 ] , sequence1 ) ) / length(sequence1);
p_2 = sum ( 1 == conv ( 1 / 2 * [1 1 ] , sequence1 ) ) /length(sequence1);
p_3 = sum ( 1 == conv ( 1 / 3 * [1 1 1 ] , sequence1 ) ) / length(sequence1);
p_4 = sum ( 1 == conv ( 1 / 4 * [1 1 1 1 ] , sequence1 ) ) / length(sequence1);
p_5 = sum ( 1 == conv ( 1 / 5 * [1 1 1 1 1 ] , sequence1 ) ) / length(sequence1);
Probability_sequence1=[p_1;p_2;p_3;p_4;p_5];
%%
%%
load sequence2.mat;
sequence2 = test_sequence;

p_1 = sum ( 1 == conv ( 1  /1 * [1 ] , sequence2 ) ) / length(sequence2);
p_2 = sum ( 1 == conv ( 1 / 2 * [1 1 ] , sequence2 ) ) /length(sequence2);
p_3 = sum ( 1 == conv ( 1 / 3 * [1 1 1 ] , sequence2 ) ) / length(sequence2);
p_4 = sum ( 1 == conv ( 1 / 4 * [1 1 1 1 ] , sequence2 ) ) / length(sequence2);
p_5 = sum ( 1 == conv ( 1 / 5 * [1 1 1 1 1 ] , sequence2 ) ) / length(sequence2);
Probability_sequence2=[p_1;p_2;p_3;p_4;p_5];
%%
%%
load sequence3.mat;
sequence3 = test_sequence;

p_1 = sum ( 1 == conv ( 1  /1 * [1 ] , sequence3 ) ) / length(sequence3);
p_2 = sum ( 1 == conv ( 1 / 2 * [1 1 ] , sequence3 ) ) /length(sequence3);
p_3 = sum ( 1 == conv ( 1 / 3 * [1 1 1 ] , sequence3 ) ) / length(sequence3);
p_4 = sum ( 1 == conv ( 1 / 4 * [1 1 1 1 ] , sequence3 ) ) / length(sequence3);
p_5 = sum ( 1 == conv ( 1 / 5 * [1 1 1 1 1 ] , sequence3 ) ) / length(sequence3);
Probability_sequence3=[p_1;p_2;p_3;p_4;p_5];
%%
%%
load sequence4.mat;
sequence4 = test_sequence;

p_1 = sum ( 1 == conv ( 1  /1 * [1 ] , sequence4 ) ) / length(sequence4);
p_2 = sum ( 1 == conv ( 1 / 2 * [1 1 ] , sequence4 ) ) /length(sequence4);
p_3 = sum ( 1 == conv ( 1 / 3 * [1 1 1 ] , sequence4 ) ) / length(sequence4);
p_4 = sum ( 1 == conv ( 1 / 4 * [1 1 1 1 ] , sequence4 ) ) / length(sequence4);
p_5 = sum ( 1 == conv ( 1 / 5 * [1 1 1 1 1 ] , sequence4 ) ) / length(sequence4);
Probability_sequence4=[p_1;p_2;p_3;p_4;p_5];
%%
%%
load sequence5.mat;
sequence5 = test_sequence;

p_1 = sum ( 1 == conv ( 1  /1 * [1 ] , sequence5 ) ) / length(sequence5);
p_2 = sum ( 1 == conv ( 1 / 2 * [1 1 ] , sequence5 ) ) /length(sequence5);
p_3 = sum ( 1 == conv ( 1 / 3 * [1 1 1 ] , sequence5 ) ) / length(sequence5);
p_4 = sum ( 1 == conv ( 1 / 4 * [1 1 1 1 ] , sequence5 ) ) / length(sequence5);
p_5 = sum ( 1 == conv ( 1 / 5 * [1 1 1 1 1 ] , sequence5 ) ) / length(sequence5);
```

```
Probability_sequence5=[p_1;p_2;p_3;p_4;p_5];
%%
%%
load sequence6.mat;
sequence6 = test_sequence;

p_1 = sum ( 1 == conv ( 1  /1 * [1 ] , sequence6 ) ) / length(sequence6);
p_2 = sum ( 1 == conv ( 1 / 2 * [1 1 ] , sequence6 ) ) /length(sequence6);
p_3 = sum ( 1 == conv ( 1 / 3 * [1 1 1 ] , sequence6 ) ) / length(sequence6);
p_4 = sum ( 1 == conv ( 1 / 4 * [1 1 1 1 ] , sequence6 ) ) / length(sequence6);
p_5 = sum ( 1 == conv ( 1 / 5 * [1 1 1 1 1 ] , sequence6 ) ) / length(sequence6);
Probability_sequence6=[p_1;p_2;p_3;p_4;p_5];

%%
Result=[Probability_sequence1 Probability_sequence2 Probability_sequence3 Probability_seq
```