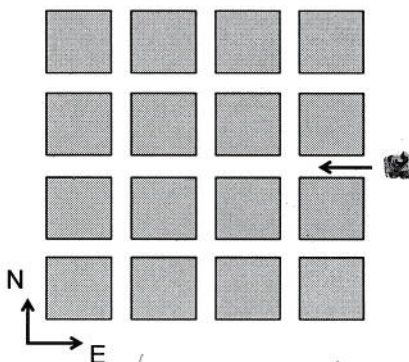


ECEn 370
Quiz #1
September 7, 2012

Name: Solution

Consider the problem of a mobile robot navigating through an rectangular grid urban terrain as shown in the Figure at the right. Suppose that at each intersection the robot continues to move in the same direction of travel half of the time, and that it turns right one fourth of the time, and that it turns left one fourth of the time. Suppose that the robot is initially heading West and that we wish to reason about its direction of travel after it encounters two intersections.



1. (40 pts) Develop a probabilistic model for this problem. Let the **outcome** of an experiment be a sequence of two directions, e.g., NW, meaning that the robot is heading North after the first intersection and West after the second intersection. What is the **sample space** Ω ? What is the set of **events**? What is the probability law?

The sample space is the set of all possible outcomes:

$$\Omega = \{NN, NE, NW, WW, WN, WS, SS, SW, SE\}$$

The set of events is all possible subsets of Ω

Probability law

$$P(NN) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

$$P(NE) = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

$$P(NW) = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

$$P(WW) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$P(WN) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$$

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$$P(SS) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

$$P(SW) = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

$$P(SE) = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

2. (30 pts) Show/argue that the probability law specified in problem 1 satisfies the three axioms of probability?

Axiom 1: $P(X) \geq 0$ - yes

Axiom 2: $P(A \cup B) = P(A) + P(B)$ if $A \cap B = \emptyset$. True since any two disjoint subsets will be the sum of prob of elements (discrete space)

Axiom 3: $P(\Omega) = \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} = \frac{1}{4} + \frac{4}{8} + \frac{4}{16} = 1$

3. (30 pts) What is the probability that the robot will be heading North after encountering two intersections?

$$P(\{NN, WN\}) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

