

ECEn 370
Quiz #10
November 16, 2012

Name: Solution

Bertsekas Problem 5.9 During each day, the probability that your computer's operating system crashes at least once is 5%, independent of every other day. Let P_{45} be the probability of at least 45 crash-free days out of the next 50 days.

- (a) Using the Binomial distribution, find the exact probability P_{45} .
- (b) Using the central limit theorem and the normal approximation to the binomial, and an approximation to P_{45} .
- (c) Using the Poisson approximation to the binomial, find an approximation to P_{45} .

a)
$$P_{45} = \sum_{k=0}^5 \binom{50}{k} (0.05)^k (0.95)^{50-k} = 0.9622$$

probability of 0, 1, 2, 3, 4, 5 crashes

b)
$$\mu = (50)(0.05) = 2.5$$

$$\sigma^2 = p(1-p) = (0.05)(0.95) = 0.0475$$

$$z = \frac{5 - \mu}{\sigma\sqrt{n}} = \frac{5 - 2.5}{\sqrt{0.0475}\sqrt{50}} = 1.622$$

Let $X = \#$ of crashes
Then $P_{45} = P(X \leq 5) \approx \Phi\left(\frac{5}{\sigma}\right)$
$$P_{45} \approx \Phi(1.622) = 0.9476$$

c)
$$P_{45} \approx \sum_{k=0}^5 \frac{\lambda^k}{k!} e^{-\lambda}$$
 where $\lambda = np = 50 \cdot 0.05 = 2.5$
$$= 0.9580$$