

ECEn 370
Quiz #11
November 30, 2012

Name: Solution

Bertsekas Problem 6.10 A fisherman catches fish according to a Poisson process with rate $\lambda = 0.6$ per hour. The fisherman will keep fishing for two hours. If he has caught at least one fish, he quits. Otherwise, he continues until he catches at least one fish.

- (a) Find the probability that he stays for more than two hours.
- (b) Find the probability that the total time he spends fishing is between two and five hours.
- (c) Find the probability that he catches at least two fish.
- (d) Find the expected number of fish that he catches.
- (e) Find the expected total fishing time, given that he has been fishing for four hours.

$$a) P(\text{stays} \geq 2 \text{ hours}) = P(0, 2) = e^{-2\lambda} \frac{(2\lambda)^0}{0!} = e^{-2\lambda} = 0.3012$$

$$b) P(\text{stays between 2-5 hours}) = P(\{\text{zero fish in 1st two hours}\} \cap \{\text{at least one fish in next three hours}\})$$

by integral of Poisson process \rightarrow $= P(\{\text{zero in } [0, 2]\}) P(\{\text{at least one in } [0, 3]\})$

$$= P(0, 2) (1 - P(0, 3))$$

$$= e^{-2\lambda} (1 - e^{-3\lambda}) = 0.2514$$

$$c) P(\{\geq 2 \text{ fish}\}) = \sum_{k=2}^{\infty} P(k, 2) = 1 - P(0, 2) - P(1, 2) = 1 - e^{-2\lambda} - (2\lambda)e^{-2\lambda}$$

$$= 0.3374$$

$$d) \text{Expected \# of fish} = \text{Expected \# in } [0, 2] + 1 \cdot P(\text{stays} \geq 2)$$

$$= 2\lambda + P(0, 2) = 1.2 + 0.3012 = 1.5012$$

e) starting at 4 hours the expected fishing time is $\frac{1}{\lambda}$

$$\therefore \text{total expected fishing time} = 4 + \frac{1}{\lambda} = 5.67 \text{ hours}$$