ECEn 370
Quiz #4
September 28, 2012

Name: Solution

Bertsekas Problem 2.4 An internet service provider uses 50 modems to serve the needs of 1000 customers. It is estimated that at a given time, each customer will need a connection with probability 0.01, independent of other customers.

(a) What is the PMF of the number of modems in use at a given time?

(b) Repeat part (a) by approximating the PMF of the number of customers that need a connection with a Poisson PMF.

(c) What is the probability that there are more customers needing a connection than there are numbers? Provide an exact, as well as an approximate formula based on the Poisson approximation of part (b).

a) $$p_X(k) = \binom{1000}{k}(0.01)^k(0.99)^{1000-k} \qquad k = 0, 1, \ldots, 49$$

$$p_X(50) = \sum_{j=50}^{1000}\binom{1000}{j}(0.01)^j(0.99)^{1000-j}$$

b) $$p_X(k) = e^{-\lambda}\frac{\lambda^k}{k!} \qquad k = 0, 1, \ldots 49 \qquad \lambda = 1000 \cdot 0.01 = 10$$

$$p_X(50) = \sum_{j=50}^{1000} e^{-\lambda}\frac{\lambda^j}{j!}$$

c)

Exact $$\sum_{j=51}^{1000}\binom{1000}{j}(0.01)^j(0.99)^{1000-j}$$

Approximate $$\sum_{j=51}^{1000} e^{-\lambda}\frac{\lambda^j}{j!}$$