ECEn 370
Quiz #5
October 4, 2012

Name: Solution

Bertsekas Problem 2.24 A stock market trader buys 100 shares of stock A and 200 shares of stock B. Let X and Y be the price changes of A and B, respectively, over a certain time period, and assume that the joint PMF of X and Y is uniform over the set of integers x and y satisfying

$$-2 \le x \le 4 \qquad -1 \le y - x \le 1.$$

(a) Find the marginal PMFs and the means of X and Y.

(b) Find the mean of the trader's profit.

for each x (7 of them) there are 3 y's $\therefore$ 21 possibilities

$\therefore$ the joint distribution is

$P_{XY}(x,y) =$

y \ x	-2	-1	0	1	2	3	4	$P_Y(y)$
5	0	0	0	0	0	0	1/21	1/21
4	0	0	0	0	0	1/21	1/21	2/21
3	0	0	0	0	1/21	1/21	1/21	3/21
2	0	0	0	1/21	1/21	1/21	0	3/21
1	0	0	1/21	1/21	1/21	0	0	3/21
0	0	1/21	1/21	1/21	0	0	0	3/21
-1	1/21	1/21	1/21	0	0	0	0	3/21
-2	1/21	1/21	0	0	0	0	0	2/21
-3	1/21	0	0	0	0	0	0	1/21
$P_X(x)$	3/21	3/21	3/21	3/21	3/21	3/21	3/21	1

$$E[X] = -2\left(\tfrac{3}{21}\right) - 1\left(\tfrac{3}{21}\right) + 0\left(\tfrac{3}{21}\right) + 1\left(\tfrac{3}{21}\right) + 2\left(\tfrac{3}{21}\right) + 3\left(\tfrac{3}{21}\right) + 4\left(\tfrac{3}{21}\right) = \frac{-6-3+0+3+6+9+12}{21} = \frac{21}{21} = 1$$

$$E[Y] = 5\left(\tfrac{1}{21}\right) + 4\left(\tfrac{2}{21}\right) + 3\left(\tfrac{3}{21}\right) + 2\left(\tfrac{3}{21}\right) + 1\left(\tfrac{3}{21}\right) + 0\left(\tfrac{3}{21}\right) - 1\left(\tfrac{3}{21}\right) - 2\left(\tfrac{2}{21}\right) - 3\left(\tfrac{1}{21}\right)$$

$$= \frac{5+8+9+6+3-3-4-3}{21} = \frac{21}{21} = 1$$

profit is:

$$E[100X + 200Y] = 100\,E[X] + 200\,E[Y] = 300 \text{ units.}$$