ECEn 370
Quiz #7
October 18, 2012

Name: Solution

Bertsekas Problem 3.10 A computer has a subroutine that can generate values of a random variable U that is uniformly distributed in the interval $[0, 1]$. Such a subroutine can be used to generate values of a continuous random variable with given CDF $F_X(x)$ as follows. If U takes a value u we let the value of X be a number x that satisfies $F_X(x) = u$. For simplicity, we assume that the given CDF is strictly increasing over the range S of values of interest, where $S = \{x \mid 0 < F_X(x) < 1\}$. This condition guarantees that for any $u \in (0, 1)$, there is a unique x that satisfies $F_X(x) = u$.

(a) Show that the CDF of the random variable X thus generated is indeed equal to the given CDF $F_X(x)$.

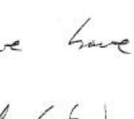
(b) Describe how this procedure can be used to simulate an exponential random variable with parameter λ.

(c) How can this procedure be generalized to simulate a discrete integer-valued random variable?

a) $P(X \le x) = P(F(X) \le F(x))$, since F is strictly increasing

$= P(U \le u)$, since $U = F(X)$

$= F_U(u)$, by definition of CDF

$= u$, $U \sim$ uniform$([0,1])$ $\Rightarrow F_U(u) =$ 

$= F(x)$

$\therefore F(x) = F_X(x)$

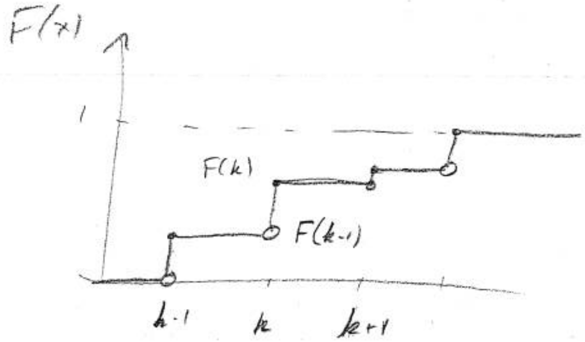
b) Since $F(x) = 1 - e^{-\lambda x}$, $x \geq 0$ is the desired CDF, we have

$u = F(x) = 1 - e^{-\lambda x} \Rightarrow e^{-\lambda x} = 1 - u \Rightarrow e^{\lambda x} = \frac{1}{1-u} \Rightarrow \lambda x = \ln\left(\frac{1}{1-u}\right)$

$\Rightarrow x = \frac{1}{\lambda} \ln\left(\frac{1}{1-u}\right)$

$\therefore$ Therefore we can sample from an exponential distribution by drawing u from a uniform distribution and letting $x = \frac{1}{\lambda} \ln\left(\frac{1}{1-u}\right)$

c) In this case F is a staircase function:



Therefore sample u from uniform $[0,1]$, then

select k s.t. $F(k-1) < u \leq F(k)$