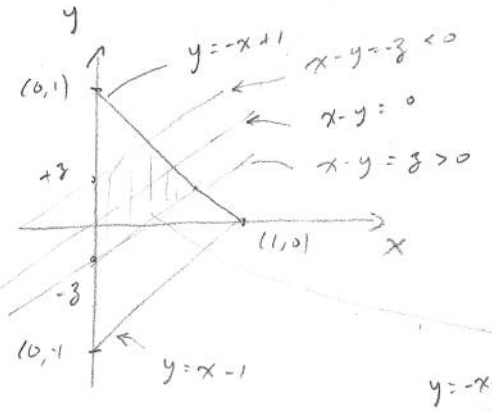
ECEn 370
Quiz #8
October 31, 2012

Name: Solution

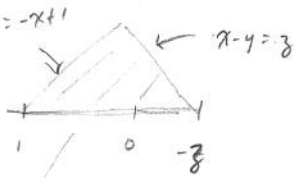
Bertsekas Problem 4.6 Let X and Y be the Cartesian coordinates of a randomly chosen point (according to a uniform PDF) in the triangle with vertices at $(0,1)$, $(0,-1)$, and $(1,0)$. Find the CDF and the PDF of $|X-Y|$.

$$F_Z(z) = P(Z \le z) = P(|X-Y| \le z) = \int_{\{x,y\}:\,|x-y|\le z} f_{XY}(x,y)\,dx\,dy = \int_{\{x,y\}:\,|x-y|\le z} dx\,dy$$

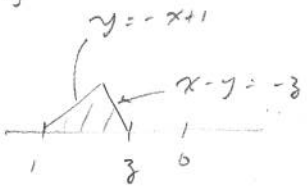


$P(|X-Y| \le z)$ $\quad z \in [0,1]$

Area is



minus



Base $= 1+z$

Height: $x-y = z$

$\Rightarrow y = x-z$

$\therefore y = -x+1 \Rightarrow x = -y+1$

$\Rightarrow x = -x+z+1$

$\Rightarrow 2x = 1+z$

$x = \frac{1}{2}(1+z)$

Area $= \frac{1}{4}(1+z)^2$

Base $= 1-z$

Height $x-y = -z \Rightarrow y = x+z$

$y = -x+1 \Rightarrow x = -y+1$

$\Rightarrow x = -x-z+1$

$\Rightarrow 2x = 1-z$

Height $= \frac{1-z}{2}$

Area $= \frac{1}{4}(1-z)^2$

$\therefore P(|X-Y| \le z) = \frac{1}{4}(1+z)^2 - \frac{1}{4}(1-z)^2 = \frac{1}{4}(1+2z+z^2-1+2z-z^2) = \frac{1}{4}4z = z$

$\therefore F_Z(z) = \begin{cases} z & 0 \le z \le 1 \\ 0 & z \le 0 \\ 1 & z \ge 1 \end{cases}$ and $f_Z(z) = \begin{cases} 1 & 0 < z < 1 \\ 0 & \text{otherwise} \end{cases}$