

ECEn 370  
Quiz #9  
November 9, 2012

Name: Solution

**Bertsekas Problem 4.22** Consider a gambler who at each gamble either wins or loses his bet with probabilities  $p$  and  $1 - p$ , independent of earlier gambles. When  $p > 1/2$ , a popular gambling system, known as the Kelly strategy, is to always bet the fraction  $2p - 1$  of the current fortune. Compute the expected fortune after  $n$  gambles, starting with  $x_0$  units and employing the Kelly strategy.

Let  $X_{k-1}$  be the amount of money available after the  $k-1$ <sup>th</sup> gamble.

$$\begin{aligned} \text{Then } E\{X_k | X_{k-1}\} &= X_{k-1} + p(2p-1)X_{k-1} - (1-p)(2p-1)X_{k-1} \\ &= [1 + 2p^2 - p - 2p + 1 + 2p^2 - p] X_{k-1} \\ &= [2 - 4p + 4p^2] X_{k-1} \end{aligned}$$

$\therefore$  By law of iterated expectation we have

$$E\{X_k\} = E\{E\{X_k | X_{k-1}\}\} = [2 - 4p + 4p^2] E\{X_{k-1}\}$$

$\wedge E\{X_0\} = x_0$ , then

$$E\{X_k\} = [2 - 4p + 4p^2]^k x_0$$