Adaptive Control and Backstepping

Consider the system

$$\dot{x} = u + \theta\varphi(x)$$ where $\varphi(\cdot)$ is a known function but θ is unknown.

Let $V = \frac{1}{2}x^2$, then

$$\dot{V} = x\dot{x} = x\left(u + \theta\varphi(x)\right)$$

If θ were known, then we would choose

$$u^d = -\theta\varphi(x) - kx^*$$

but since it is not we will use the "certainty equivalence" controller

$$u = -\hat{\theta}\varphi(x) - kx$$

where $\hat{\theta}$ is an estimate of θ. The result is

$$\begin{aligned}
\dot{V} &= x\left(u^d + \theta\varphi(x) + u - u^d\right) \\
&= -kx^2 + x\left(u - u^d\right) \\
&= -kx^2 + x\left(-\hat{\theta}\varphi - kx + \theta\varphi(x) + kx\right) \\
&= -kx^2 + (\theta - \hat{\theta})\, x\, \varphi(x)
\end{aligned}$$

Let $\tilde{\theta} = \theta - \hat{\theta}$ and note that $\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$

And augment the Lyapunov function as

$$V = \frac{1}{2}x^2 + \frac{1}{2\gamma}\tilde{\theta}^2$$

Then

$$\dot{V} = -kx^2 + \tilde{\theta}\, x\varphi(x) + \frac{1}{\gamma}\tilde{\theta}\dot{\tilde{\theta}}$$

$$= -kx^2 + \tilde{\theta}\left(x\,\varphi(x) - \frac{1}{\gamma}\dot{\hat{\theta}}\right)$$

If we let $\dot{\hat{\theta}} = \gamma x \varphi(x)$, then

$$\dot{V} = -kx^2 \leq 0$$

We can now argue that $x \to 0$ ~~by the~~ using Barbalat's Lemma.

Note that $\tilde{\theta}$ does not necessarily go to zero.

Now suppose that we have the system

$$\dot{x}_1 = \theta \varphi(x_1) + x_2$$
$$\dot{x}_2 = u$$

Following the same steps, let

$V_1 = \frac{1}{2} x_1^2$ then

$$\dot{V}_1 = x_1 \left(\theta \varphi(x_1) + x_2 \right)$$

Thinking of x_2 as the control, let

$$x_2^d = -\hat{\theta} \varphi - k_1 x_1$$

where $\hat{\theta}$ is an estimate of θ. Then

$$\begin{aligned}
\dot{V}_1 &= x_1 \left(\theta \varphi(x_1) + x_2^d + x_2 - x_2^d \right) \\
&= x_1 \left(\theta \varphi(x_1) - \hat{\theta} \varphi(x_1) - k_1 x_1 + x_2 - x_2^d \right) \\
&= -k_1 x_1^2 + (\theta - \hat{\theta}) x_1 \varphi(x_1) + x_1 (x_2 - x_2^d)
\end{aligned}$$

Defining $\tilde{\theta} \triangleq \theta - \hat{\theta} \Rightarrow \dot{\tilde{\theta}} = \dot{\theta} - \dot{\hat{\theta}} = -\dot{\hat{\theta}}$

$z \triangleq x_2 - x_2^d \Rightarrow \dot{z} = \dot{x}_2 - \dot{x}_2^d = u - \dot{x}_2^d$

gives

$$\dot{V}_1 = -k_1 x_1^2 + \tilde{\theta} x_1 \varphi(x_1) + x_1 z$$

Revise V_1 as

$$V_2 = \frac{1}{2} x_1^2 + \frac{1}{2\gamma} \tilde{\theta}^2$$

to get

$$\dot{V}_2 = -k_1 x_1^2 + \tilde{\theta} x_1 \varphi(x_1) + x_1 z + \tilde{\theta}\dot{\tilde{\theta}}$$

$$= -k_1 x_1^2 + \tilde{\theta}\left(x_1 \varphi(x_1) - \frac{1}{\gamma}\dot{\hat{\theta}} \right) + x_1 z$$

Select $\dot{\hat{\theta}} = \gamma x_1 \varphi(x_1)$ to get

$$\dot{V}_2 = -k_1 x_1^2 + x_1 z$$

Now revise V_2 as

$$V_3 = 1/2\, x_1^2 + \frac{1}{2\gamma}\tilde{\theta}^2 + \frac{1}{2} z^2$$

$$\dot{V}_3 = -k_1 x_1^2 + x_1 z + z\dot{z}$$

$$= -k_1 x_1^2 + z\left(x_1 + u - \dot{x}_2^d \right)$$

Let $u = -x_1 + \dot{x}_2^d - k_2 z$ to get

$$\dot{V}_3 = -k_1 x_1^2 - k_2 z^2$$

which renders the system GAS without knowing θ!

Adaptive Trajectory Tracking for Unmanned Air Vehicles

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Abstract

This is the abstract.

1 Introduction

This is the introduction.

2 Problem Description

Let $\mathbf{z} = (z_x, z_y)^T$ be the inertial position of the UAV and let ψ its heading angle. Also, let v be ground speed velocity of the UAV.

Assuming that an autopilot has been designed for the UAV such that the response of the controlled aircraft to commands in both heading and velocity, then the kinematic equations of motion become

$$\begin{aligned} \dot{\mathbf{z}} &= v\mathbf{s}(\psi) \\ \dot{\psi} &= \alpha_\psi (\psi^c - \psi) \quad (1) \\ \dot{v} &= \alpha_v (v^c - v), \quad (2) \end{aligned}$$

where

$$\mathbf{s}(\psi) = \begin{pmatrix} \cos(\psi) \\ \sin(\psi) \end{pmatrix}.$$

The autopilot constants α_ψ and α_v are unknown positive constants. The input signals ψ^c and v^c are the autopilot commands.

The objective is to asymptotically track the output of the trajectory that is produced by the dynamic trajectory smoother (DTS) shown in Figure ??. The DTS produces trajectories which satisfy the following equations:

$$\begin{aligned} \dot{\mathbf{z}}^d &= v^d \mathbf{s}(\psi^d) \\ \dot{\psi}^d &= u \\ \dot{v}^d &= 0, \end{aligned} \quad (3)$$

where u is a signal produced by the DTS and is known to the ATT.

3 Adaptive Trajectory Tracker

Step 1. Tracking.

The first step is to assume that $v\mathbf{s}(\psi)$ is a control input vector and to use this pseudo-control to drive the tracking error $\|\mathbf{z} - \mathbf{z}^d\|$ to zero.

Consider the Lyapunov function candidate

$$V_1 = \frac{1}{2} \left\|\mathbf{z} - \mathbf{z}^d\right\|^2. \quad (4)$$

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Differentiating we obtain

$$\begin{aligned} \dot{V}_1 &= \left(\mathbf{z} - \mathbf{z}^d\right)^T \left(\dot{\mathbf{z}} - \dot{\mathbf{z}}^d\right) \\ &= \left(\mathbf{z} - \mathbf{z}^d\right)^T \left(v\mathbf{s}(\psi) - v^d\mathbf{s}(\psi^d)\right). \end{aligned} \quad (5)$$

If $\xi = v\mathbf{s}(\psi)$ were the control signal, then we would set

$$\xi = v^d\mathbf{s}(\psi^d) - K_1 \left(\mathbf{z} - \mathbf{z}^d\right), \quad (6)$$

where $K_1 > 0$, to obtain

$$\dot{V}_1 = -\left(\mathbf{z} - \mathbf{z}^d\right)^T K_1 \left(\mathbf{z} - \mathbf{z}^d\right) < 0.$$

Step 2. Backstepping.

Unfortunately, $\xi = v\mathbf{s}(\psi)$ is not a true control input, so in reality we have

$$\dot{V}_1 = \left(\mathbf{z} - \mathbf{z}^d\right)^T \left(v\mathbf{s}(\psi) - v^d\mathbf{s}(\psi^d) + \xi - \xi\right), \quad (7)$$

where we have added and subtracted ξ from Equation (5). Plugging (??) into (7) gives

$$\begin{aligned} \dot{V}_1 = &-\left(\mathbf{z} - \mathbf{z}^d\right)^T K_1 \left(\mathbf{z} - \mathbf{z}^d\right) \\ &+ \left(\mathbf{z} - \mathbf{z}^d\right)^T \left(v\mathbf{s}(\psi) - v^d\mathbf{s}(\psi^d) + K_1(\mathbf{z} - \mathbf{z}^d)\right). \end{aligned}$$

Defining

$$\mathbf{q}_1 \triangleq v\mathbf{s}(\psi) - v^d\mathbf{s}(\psi^d) + K_1(\mathbf{z} - \mathbf{z}^d), \quad (8)$$

gives

$$\dot{V}_1 = -\left(\mathbf{z} - \mathbf{z}^d\right)^T K_1 \left(\mathbf{z} - \mathbf{z}^d\right) + \left(\mathbf{z} - \mathbf{z}^d\right)^T \mathbf{q}_1.$$

Next consider the modified Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2}\mathbf{q}_1^T\mathbf{q}_1,$$

where

$$\dot{V}_2 = -\left(\mathbf{z} - \mathbf{z}^d\right)^T K_1 \left(\mathbf{z} - \mathbf{z}^d\right) + \left(\mathbf{z} - \mathbf{z}^d\right)^T \mathbf{q}_1 + \dot{\mathbf{q}}_1^T\mathbf{q}. \quad (9)$$

Differentiating (8) and plugging into (9) gives

$$\begin{aligned} \dot{V}_2 = &-\left(\mathbf{z} - \mathbf{z}^d\right)^T K_1 \left(\mathbf{z} - \mathbf{z}^d\right) \\ &+ \mathbf{q}_1^T \Big[\left(\mathbf{z} - \mathbf{z}^d\right) + \dot{v}\mathbf{s}(\psi) + v\mathbf{s}'(\psi)\dot{\psi} - \dot{v}^d\mathbf{s}(\psi^d) \\ &\qquad - v^d\mathbf{s}'(\psi^d)\dot{\psi}^d + K_1\dot{\mathbf{z}} - K_1\dot{\mathbf{z}}^d\Big], \end{aligned}$$

where

$$\mathbf{s}'(\psi) \triangleq \frac{d}{d\psi}\begin{pmatrix}\cos(\psi)\\ \sin(\psi)\end{pmatrix} = \begin{pmatrix}-\sin(\psi)\\ \cos(\psi)\end{pmatrix}.$$

Using (1) and (3) we obtain

$$\begin{aligned}\dot{V}_2 = &-\left(\mathbf{z}-\mathbf{z}^d\right)^T K_1\left(\mathbf{z}-\mathbf{z}^d\right)\\ &+\mathbf{q}_1^T\left[\left(\mathbf{z}-\mathbf{z}^d\right)+\alpha_v\left(v^c-v\right)\mathbf{s}(\psi)\right.\\ &\quad+\alpha_\psi\left(\psi^c-\psi\right)v\mathbf{s}'(\psi)-v^d u\mathbf{s}'(\psi^d)\\ &\quad\left.+K_1 v\mathbf{s}(\psi)-K_1 v^d\mathbf{s}(\psi^d)\right]\\ =&-\left(\mathbf{z}-\mathbf{z}^d\right)^T K_1\left(\mathbf{z}-\mathbf{z}^d\right)\\ &+\mathbf{q}_1^T\left[\left(\mathbf{s}(\psi)\quad v\mathbf{s}'(\psi)\right)\begin{pmatrix}\alpha_v\left(v^c-v\right)\\ \alpha_\psi\left(\psi^c-\psi\right)\end{pmatrix}+\left(\mathbf{z}-\mathbf{z}^d\right)\right.\\ &\quad\left.-v^d u\mathbf{s}'(\psi^d)+K_1\left(v\mathbf{s}(\psi)-v^d\mathbf{s}(\psi^d)\right)\right].\end{aligned}$$

Defining

$$R(\psi) \triangleq \left(\mathbf{s}(\psi)\quad v\mathbf{s}'(\psi)\right) = \begin{pmatrix}\cos(\psi) & -v\sin(\psi)\\ \sin(\psi) & v\cos(\psi)\end{pmatrix}$$

$$\mathbf{q}_2 = \left(\mathbf{z}-\mathbf{z}^d\right)-v^d u\mathbf{s}'(\psi^d)+K_1\left(v\mathbf{s}(\psi)-v^d\mathbf{s}(\psi^d)\right),$$

we obtain

$$\begin{aligned}\dot{V}_2 = &-\left(\mathbf{z}-\mathbf{z}^d\right)^T K_1\left(\mathbf{z}-\mathbf{z}^d\right)\\ &+\mathbf{q}_1^T\left[R(\psi)\begin{pmatrix}\alpha_v & 0\\ 0 & \alpha_\psi\end{pmatrix}\begin{pmatrix}v^c-v\\ \psi^c-\psi\end{pmatrix}+\mathbf{q}_2\right].\end{aligned} \quad (10)$$

If α_v and α_ψ were known then we could let

$$\begin{pmatrix}v^c-v\\ \psi^c-\psi\end{pmatrix} = \begin{pmatrix}\frac{1}{\alpha_v} & 0\\ 0 & \frac{1}{\alpha_v}\end{pmatrix}R^{-1}(\psi)\left[-\mathbf{q}_2-K_2\mathbf{q}_1\right], \quad (11)$$

which would render Equation (10)

$$\dot{V}_2 = -\left(\mathbf{z}-\mathbf{z}^d\right)^T K_1\left(\mathbf{z}-\mathbf{z}^d\right)-\mathbf{q}_1^T K_2\mathbf{q}_1,$$

which is negative definite if $K_2 > 0$.

Step 3. Adaptation.

Unfortunately we do not know α_v and α_ψ so we cannot implement Equation (11). Instead we will use

$$\begin{pmatrix}v^c-v\\ \psi^c-\psi\end{pmatrix} = \begin{pmatrix}\frac{1}{\hat{\alpha}_v} & 0\\ 0 & \frac{1}{\hat{\alpha}_v}\end{pmatrix}R^{-1}(\psi)\left[-\mathbf{q}_2-K_2\mathbf{q}_1\right], \quad (12)$$

where $\hat{\alpha}_v$ and $\hat{\alpha}_\psi$ are the current estimates of α_v and α_ψ. Equation (10) therefore becomes

$$\begin{aligned}\dot{V}_2 = &-\left(\mathbf{z}-\mathbf{z}^d\right)^T K_1\left(\mathbf{z}-\mathbf{z}^d\right)\\ &+\mathbf{q}_1^T\left[R(\psi)\begin{pmatrix}\alpha_v & 0\\ 0 & \alpha_\psi\end{pmatrix}\begin{pmatrix}\frac{1}{\hat{\alpha}_v} & 0\\ 0 & \frac{1}{\hat{\alpha}_\psi}\end{pmatrix}R^{-1}(\psi)\left[-\mathbf{q}_2-K_2\mathbf{q}_1\right]\right.\\ &+\mathbf{q}_2\\ &+R(\psi)\begin{pmatrix}\alpha_v & 0\\ 0 & \alpha_\psi\end{pmatrix}\begin{pmatrix}\frac{1}{\alpha_v} & 0\\ 0 & \frac{1}{\alpha_\psi}\end{pmatrix}R^{-1}(\psi)\left[-\mathbf{q}_2-K_2\mathbf{q}_1\right]\\ &\left.-R(\psi)\begin{pmatrix}\alpha_v & 0\\ 0 & \alpha_\psi\end{pmatrix}\begin{pmatrix}\frac{1}{\alpha_v} & 0\\ 0 & \frac{1}{\alpha_\psi}\end{pmatrix}R^{-1}(\psi)\left[-\mathbf{q}_2-K_2\mathbf{q}_1\right]\right],\end{aligned} \quad (13)$$

where we have added and subtracted Equation (11). After some algebra we obtain

$$\begin{aligned}\dot{V}_2 = &-\left(\mathbf{z}-\mathbf{z}^d\right)^T K_1\left(\mathbf{z}-\mathbf{z}^d\right)-\mathbf{q}_1^T K_2\mathbf{q}_1\\ &+\mathbf{q}_1^T R(\psi)\begin{pmatrix}\alpha_v & 0\\ 0 & \alpha_\psi\end{pmatrix}\,\mathrm{diag}\left(R^{-1}(\psi)\left[-\mathbf{q}_2-K_1\mathbf{q}_1\right]\right)\\ &\begin{pmatrix}\frac{1}{\hat{\alpha}_v}-\frac{1}{\alpha_v}\\ \frac{1}{\hat{\alpha}_\psi}-\frac{1}{\alpha_\psi}\end{pmatrix},\end{aligned} \quad (14)$$

where

$$\mathrm{diag}\left(\begin{pmatrix}w_1\\ w_2\end{pmatrix}\right) = \begin{pmatrix}w_1 & 0\\ 0 & w_2\end{pmatrix}.$$

Defining $\boldsymbol{\theta} = \left(\frac{1}{\alpha_v}\quad\frac{1}{\alpha_\psi}\right)^T$ and $\hat{\boldsymbol{\theta}} = \left(\frac{1}{\hat{\alpha}_v}\quad\frac{1}{\hat{\alpha}_\psi}\right)^T$, and noting that diagonal matrices commute gives

$$\begin{aligned}\dot{V}_2 = &-\left(\mathbf{z}-\mathbf{z}^d\right)^T K_1\left(\mathbf{z}-\mathbf{z}^d\right)-\mathbf{q}_1^T K_2\mathbf{q}_1\\ &+\mathbf{q}_1^T R(\psi)\begin{pmatrix}\alpha_v & 0\\ 0 & \alpha_\psi\end{pmatrix}\,\mathrm{diag}\left(R^{-1}(\psi)\left[-\mathbf{q}_2-K_1\mathbf{q}_1\right]\right)\\ &\begin{pmatrix}\alpha_v & 0\\ 0 & \alpha_\psi\end{pmatrix}\left(\hat{\boldsymbol{\theta}}-\boldsymbol{\theta}\right).\end{aligned} \quad (15)$$

Consider the Lyapunov function candidate

$$V_3 = V_2+\alpha\gamma\left(\hat{\boldsymbol{\theta}}-\boldsymbol{\theta}\right)^T\begin{pmatrix}\alpha_v & 0\\ 0 & \alpha_\psi\end{pmatrix}\left(\hat{\boldsymbol{\theta}}-\boldsymbol{\theta}\right).$$

Differentiating and using Equation (15) gives

$$\begin{aligned}\dot{V}_3 = &-\left(\mathbf{z}-\mathbf{z}^d\right)^T K_1\left(\mathbf{z}-\mathbf{z}^d\right)-\mathbf{q}_1^T K_2\mathbf{q}_1\\ &+\mathbf{q}_1^T R(\psi)\begin{pmatrix}\alpha_v & 0\\ 0 & \alpha_\psi\end{pmatrix}\,\mathrm{diag}\left(R^{-1}(\psi)\left[-\mathbf{q}_2-K_1\mathbf{q}_1\right]\right)\\ &\begin{pmatrix}\alpha_v & 0\\ 0 & \alpha_\psi\end{pmatrix}\left(\hat{\boldsymbol{\theta}}-\boldsymbol{\theta}\right)+\gamma\left(\dot{\hat{\boldsymbol{\theta}}}-\dot{\boldsymbol{\theta}}\right)^T\begin{pmatrix}\alpha_v & 0\\ 0 & \alpha_\psi\end{pmatrix}\left(\hat{\boldsymbol{\theta}}-\boldsymbol{\theta}\right).\end{aligned} \quad (16)$$

Noting that $\dot{\boldsymbol{\theta}} \equiv 0$, we can choose the parameter update law to cancel the last two terms in (16), i.e.,

$$\dot{\hat{\boldsymbol{\theta}}} = \frac{1}{\gamma}\,\mathrm{diag}\left(R^{-1}(\psi)\left[\mathbf{q}_2+K_1\mathbf{q}_1\right]\right)R^T(\psi)\mathbf{q}_1.$$

Therefore Equation (16) becomes

$$\dot{V}_3 = -\left(\mathbf{z}-\mathbf{z}^d\right)^T K_1\left(\mathbf{z}-\mathbf{z}^d\right)-\mathbf{q}_1^T K_2\mathbf{q}_1 < 0.$$

Therefore we have proved the following theorem.

Theorem 3.1 *The adaptive control strategy given by the dynamic system*

$$\dot{\hat{\boldsymbol{\theta}}} = \frac{1}{\gamma}\,diag\left(R^{-1}(\psi)\left[\mathbf{q}_2+K_1\mathbf{q}_1\right]\right)R^T(\psi)\mathbf{q}_1$$

$$\begin{pmatrix}v^c\\ \psi^c\end{pmatrix} = \begin{pmatrix}v\\ \psi\end{pmatrix}\begin{pmatrix}\frac{1}{\hat{\alpha}_v} & 0\\ 0 & \frac{1}{\hat{\alpha}_v}\end{pmatrix}R^{-1}(\psi)\left[-\mathbf{q}_2-K_2\mathbf{q}_1\right],$$

where

$$R(\psi) = \begin{pmatrix} \cos(\psi) & -v\sin(\psi) \\ \sin(\psi) & v\cos(\psi) \end{pmatrix}$$
$$\mathbf{q}_1 = v\mathbf{s}(\psi) - v^d\mathbf{s}(\psi^d) + K_1(\mathbf{z} - \mathbf{z}^d),$$
$$\mathbf{q}_2 = (\mathbf{z} - \mathbf{z}^d) - v^d u\mathbf{s}'(\psi^d) + K_1\left(v\mathbf{s}(\psi) - v^d\mathbf{s}(\psi^d)\right),$$

and $\mathbf{z}^d$, ψ^d, v^d, u *come from Equation* (3), *and* $K_1 > 0$, $K_2 > 0$, *and* $\gamma > 0$ *are control gains, renders the system* (1) *uniformly asymptotically stable.*

4 Simulation Results

5 Conclusions

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