Sliding Mode

Motivating example:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = h(x) + g(x)u$$

where $h(x)$ and $g(x)$ are unknown but

$$g(x) \geq g_0 > 0$$

Step 1: Choose a sliding manifold:

Let $s = a_1 x_1 + x_2 = 0$

Note that when $s = 0$, $x_2 = \dot{x}_1 = a x_1$

which implies that $x_1 \to 0$, if $\dot{x}_1$ also goes to zero, then $x_2 \to 0$

Step 2 pick u to drive all trajectories to $s = 0$

Note that $\dot{s} = a_1 \dot{x}_1 + \dot{x}_2$

$$= a_1 x_2 + h + gu$$

Let $V = \frac{1}{2} s^2$

then

$$\dot{V} = s\dot{s} = s(a_1 x_2 + h + gu)$$
$$= sg\left[\frac{a_1 x_2 + h}{g} + u\right]$$

Suppose $\left|\frac{a_1 x_2 + h(x)}{g(x)}\right| \leq \rho(x) \quad \forall x \in \mathbb{R}^2$

Then

$$\dot{V} \leq g(x)\,|s|\left|\frac{a_1 x_2 + h}{g}\right| + g\,s\,u$$

$$\leq g(x)\,|s|\,\rho(x) + g(x)\,s\,u$$

Let $u = -\beta(x)\,\mathrm{sgn}(s)$

where $\beta(x) \geq \rho(x) + \beta_0$ then

$$\dot{V} \leq g(x)|s|\rho(x) - g(x)|s|\rho(x) - g(x)|s|\beta_0$$

$$= -g(x)\beta_0 |s|$$

$$\leq -g_0 \beta_0 |s|$$

$\therefore$ Trajectories starting outside of S, converge to S.

In fact they converge to S in finite time as shown by the following:

Let $W = \sqrt{2V}$, then

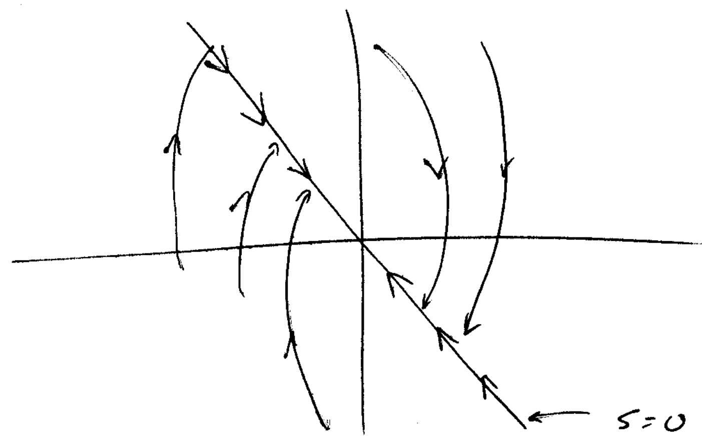
$$\dot{W} = \frac{\dot{V}}{\sqrt{2V}} \leq \frac{-g_0\beta_0|s|}{\sqrt{s^2}} = \frac{-g_0\beta_0|s|}{|s|} = -g_0\beta_0$$

$$\Rightarrow W(s(t)) \leq W(s(0)) - g_0\beta_0 t$$

$s(t) = 0$ in finite time, namely

$$t^* \leq \frac{W(s(0))}{g_0\beta_0}$$

Phase portrait



The set $\{x : s = 0\}$ is called the "<u>sliding manifold</u>"

The motion before reaching the sliding manifold is called the "reaching phase."

The motion on the sliding manifold is called the "sliding phase."

The controller $u = -\beta(x)\,\mathrm{sgn}(s)$ is the sliding mode control.

Advantages: does not require exact knowledge of $h(x)$ and $g(x)$, i.e. robustness

<u>Region of Attraction</u>

If we know that $\left| \frac{a_1 x_2 + h(x)}{g(x)} \right| \le k_1$

Then we can let $u = -k \,\text{sgn}(s)$, $k > k_1$

Consider the set [illegible] $\hat{S} = \{ s : |s| \le c \}$

The condition $s\dot{s} \le 0$ implies $\hat{S}$ is positively invariant

Since $s = a_1 x_1 + x_2 \Rightarrow x_2 = -a_1 x_1 + s$

$$\therefore \quad \dot{x}_1 = x_2 = -a_1 x_1 + s$$

Setting $V_1 = \frac{1}{2} x_1^2$ gives

$$\dot{V}_1 = x_1 \dot{x}_1 = -a_1 x_1^2 + x_1 s$$

$$\le -a_1 x_1^2 + |x_1||s|$$

$$\le -a_1 x_1^2 + |x_1| c$$

$$\le 0 \quad \text{if} \quad |x_1| \ge \frac{c}{a_1}$$

$$\therefore \quad |x_1(0)| \le \frac{c}{a_1} \Rightarrow |x_1(t)| \le \frac{c}{a_1}$$

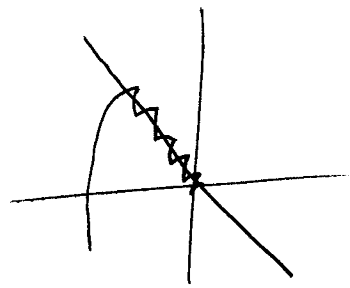
$\therefore \quad \Omega = \{ |x_1| \le \frac{c}{a_1}, \ |s| \le c \}$ is positively invariant

if $\left| \frac{a_1 x_2 + h(x)}{g(x)} \right| \le k_1 \quad \forall x \in \Omega$

Since $x_1(t) \to 0$ and $s(t) \to 0 \Rightarrow x_2(t) \to 0$

we have that every ~~[illegible]~~ trajectory starting in Ω goes the the origin.

Disadvantage: chattering - since can't switch instantaneously, the state will chatter around the sliding surface.



Example: inverted pendulum

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \underbrace{-(g_0/l)\sin(x_1+\delta_1) - (k_0/m)\,x_2}_{h(x)} + \underbrace{\left(\frac{1}{ml^2}\right)}_{g(x)} u$$

Note

$$\left|\frac{a_1 x_2 + h(x)}{g(x)}\right| = \left| m l^2 a_1 x_2 - m g_0 l \sin(x_1+\delta) - k_0 l^2 x_2 \right|$$

$$= \left| l^2 (m a_1 - k_0)\, x_2 - m g_0 l \sin(x_1+\delta) \right|$$

$$\le l^2 |m a_1 - k_0|\,|x_2| + m g_0 l$$

pick $a_1 = 1$ and let $|x_1| \le \pi \doteq c$

Then on the set $\{ |x_1| \le \pi,\ |s| = |x_1 + x_2| < \pi \}$

we have

$$\left|\frac{a x_1 + h(x)}{g(x)}\right| \le l^2 |m - k_0|\,|2\pi| + m g_0 l$$

[illegible]

$$\le \bar{l}^2 |\bar{m} - \underline{k}_0|(2\pi) + \bar{m} g_0 \bar{l}$$

where $\underline{l} \le l \le \bar{l}$, $\underline{m} \le m \le \bar{m}$, $\underline{k}_0 \le k \le \bar{k}_0$

<u>Example</u>

$$\ddot{y} = h(y,\dot{y}) + g(y)u$$

where we know $\hat{h}(y,\dot{y})$ and $\hat{g}(y)$ s.t.

$$|\hat{h} - h| < \bar{h}$$
$$|\hat{g} - g| < \bar{g}$$

Let $s = \dot{y} + a_1 y$ where $a_1 > 0$

Note that if $s = 0$ then $\dot{y} = -a_1 y \Rightarrow y(t) = e^{-a_1 t} y(0)$

Let
$$x_1 = y$$
$$x_2 = \dot{y}$$

$$\Rightarrow \quad \dot{x}_1 = x_2$$
$$\dot{x}_2 = h(x) + g(x)u$$

where $s = x_2 + a_1 x_1$

let $V = \frac{1}{2}s^2$

$$\Rightarrow \dot{V} = s\dot{s} = s(\dot{x}_2 + a_1\dot{x}_1)$$
$$= s(h(x) + g(x)u + a_1 x_2)$$
$$= s(\hat{h} + (h-\hat{h}) + \hat{g}u + (g-\hat{g})u + a_1 x_2)$$

Let $u = -\frac{[a_1 x_2 + \hat{h}]}{\hat{g}} + v$

$$\Rightarrow \dot{V} = s\left[(h-\hat{h}) + \hat{g}v - \frac{(g-\hat{g})}{\hat{g}}(a x_2 + \hat{h}) + (g-\hat{g})v\right]$$
$$= s\left[(h-\hat{h}) - \frac{(g-\hat{g})}{\hat{g}}(a x_2 + \hat{h}) + gv\right]$$

Example

$$\ddot{y} = h(y,\dot{y}) + u$$

where we know $\hat{h}(y,\dot{y})$ and

$|h - \hat{h}| \le \bar{h}$

Let $s = \dot{y} + a_1 y$ where $a_1 > 0$.

Note that if $s = 0$ then $\dot{y} = -a_1 y \Rightarrow y(t) = e^{-a_1 t} y(0)$

Let $x_1 = y$
$x_2 = \dot{y}$

$\Rightarrow$ $\dot{x}_1 = x_2$
$\dot{x}_2 = h + u = \hat{h} + (h - \hat{h}) + u$

Let $V = \frac{1}{2} s^2 \Rightarrow \dot{V} = s\dot{s} = s(\hat{h} + (h-\hat{h}) + u + a_1 x_2)$

let $u = -\hat{h} - a_1 x_2 + v$

then $\dot{V} = s(h - \hat{h} + u)$

$\le |s||h-\hat{h}| + su$

$\le |s|\bar{h} + su$

Let $u = -k\,\mathrm{sgn}(s)$

$\Rightarrow \dot{V} = |s|\bar{h} - k|s|$

$= -(k - \bar{h})|s|$

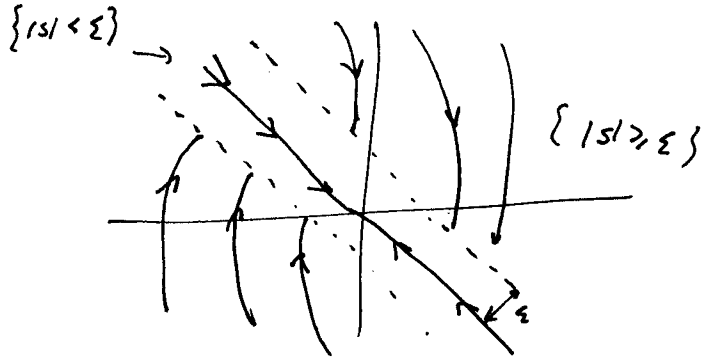
$\therefore$ if $k > \bar{h}$ then $\dot{V} < 0$

& converge to sliding mode in finite time.

What about chattering ?

Basic idea :

Instead of switching exactly on s, place a buffer around s



Instead of using

$$u = -\beta(x)\,\mathrm{sgn}(s)$$

use

$$u = -\beta(x)\,\mathrm{sat}\left(\frac{s}{\varepsilon}\right) \quad \text{where} \quad \mathrm{sat}(y) = \begin{cases} y & \text{if } |y| \le 1 \\ \mathrm{sgn}(y) & \text{if } |y| > 1 \end{cases}$$

In the set $\{|s| \geq \varepsilon\}$ we again have

$$\dot{V} \leq -g_0 \beta_0 |s|$$

with finite time convergence to the set $\{|s| < \varepsilon\}$

Inside the set $\{|s| < \varepsilon\}$ we have that

$$\dot{x}_1 = x_2 = -a x_1 + s$$

$\therefore$ The derivative of $V_1 = \frac{1}{2} x_1^2$

so
$$\begin{aligned} \dot V_1 &= x_1 \dot x_1 \\ &= -a_1 x_1^2 + x_1 s \\ &\le -a_1 x_1^2 + |x_1||s| \\ &\le -a_1 x_1^2 + |x_1| \varepsilon \\ &= -(1-\theta) a_1 x_1^2 + \left(-\theta_1 a_1 x_1^2 + |x_1| \varepsilon \right) \qquad 0 < \theta_1 < 1 \\ &\le -(1-\theta_1) a_1 x_1^2 \qquad \text{if} \quad |x_1| \ge \frac{\varepsilon}{a_1 \theta_1} \end{aligned}$$

$$\therefore \quad x \to \Omega_\varepsilon = \left\{ |x_1| \le \frac{\varepsilon}{a_1 \theta_1} \,,\; |s| \le \varepsilon \right\}$$

In general, the origin may not be stabilized, but ε can be made arbitrarily small.

The origin may be stabilized but this must be investigated independently.

Generalization

Given a nonlinear system $\dot{x} = f(x,u)$, suppose that we can find a change of variables to put the system in the form

$$\dot{\eta} = f_a(\eta, \xi)$$
$$\dot{\xi} = f_b(\eta, \xi) + G(x)E(x)u + \delta(t,x,u)$$

where f_a, f_b, E – known, E - invertable

G, δ – unknown

G – diagonal + bounded away from zero

Also, assume that if we think of ξ as an input to the first equation, that we can find a stabilizing control $\phi(\eta)$ s.t.

$\dot{\eta} = f_a(\eta, \phi(\eta))$ is asymptotically stable.

Since we desire $\xi = \phi(\eta)$, let

$$S \triangleq \xi - \phi(\eta)$$

Then $\dot{s} = \dot{\xi} - \frac{\partial \phi}{\partial \eta}\dot{\eta}$

$$= f_b + GEu + \delta - \frac{\partial \phi}{\partial \eta} f_a$$

Let $u = E^{-1}\left\{ -\hat{G}^{-1}\left[f_b - \frac{\partial\phi}{\partial\eta} f_a \right] + v \right\}$

where $\hat{G}$ is the nominal model of G

Since $G(x) = \begin{pmatrix} g_1(x) & & 0 \\ & \ddots & \\ 0 & & g_p(x) \end{pmatrix}$ is diagonal we have

$$\dot{s}_i = g_i(x)\, v_i + \Delta_i(t, x, u)$$

where Δ_i is all of the left over junk.

Assume that we know bounds on Δ_i, i.e.

$$\left| \frac{\Delta_i(t,x,u)}{g_i(x)} \right| \le \rho(x) + k_0 \|v\|_\infty \qquad k_0 \in [0,1)$$

Let $V_i = \frac{1}{2} s_i^2$ then

$$\begin{aligned} \dot{V}_i &= s_i \dot{s}_i \\ &= s_i (g_i v + \Delta_i) \\ &\le g_i \left\{ s_i v_i + |s_i| \left| \frac{\Delta_i}{g_i} \right| \right\} \\ &\le g_i \left\{ s_i v_i + |s_i| \left[\rho + k_0 \|v\|_\infty \right] \right\} \end{aligned}$$

Take $v_i = -\beta(x)\, \mathrm{sgn}(s_i)$ then

$$\begin{aligned} \dot{V}_i &\le g_i \left\{ -\beta(x)|s_i| + |s_i| \left[\rho + k_0 \|v\|_\infty \right] \right\} \\ &\le g_i \left\{ -\beta(x) + \rho + k_0 \beta \right\} |s_i| \\ &= g_i \left[-(1-k_0)\beta + \rho \right] |s_i| \end{aligned}$$

Example 1

Given

1st order linear system

$$\dot{y} = ay + bu$$

a, b are not known precisely.

Assume we have a nominal model

$\hat{a}$ $\hat{b} > 0$ and that we know that

~~$|a - \hat{a}| \leq |\bar{a}|$~~

$$a = \hat{a} + \delta a \quad \text{where} \quad |\delta a| \leq \bar{a}$$

$$b = \hat{b} + \delta b \quad \text{where} \quad 0 < \underline{b} \leq \delta b \leq \bar{b}$$

Similar to the adaptive control problem, assume that our objective is to follow a reference model

$$\dot{y}_m = a_m y_m + b_m r$$

Let $\tilde{y} = y - y_m$

Find a sliding mode controller that drives $\tilde{y} \to 0$.

The ~~stde~~ sliding surface is one dimension less then the ~~degree~~ order of the system. Since the original system has order 1, the sliding surface will have dimension = 0.

Let $s = \tilde{y} = y - y_m$.

Also let $V = \frac{1}{2} s^2$, then

$\dot{V} = s\dot{s}$ ~~to~~ where

$\dot{s} = \dot{\tilde{y}} = \dot{y} - \dot{y}_m = ay + bu - a_m y_m - b_m r$

So

$$\dot{V} = s\left[ay + bu - a_m y_m - b_m r \right]$$

$$= s\left[\hat{a}y + \delta a\, y + bu - a_m y_m - b_m r \right]$$

$$= bs\left[\frac{\hat{a}y - a_m y_m - b_m r}{b} + u + \frac{\delta a}{b} y \right]$$

Let $u = -\left[\frac{\hat{a}y - a_m y_m - b_m r}{\hat{b}} \right] + v$

Then

$$\dot{V} = bs\left\{ (\hat{a}y - a_m y_m - b_m r)\left(\frac{1}{b} - \frac{1}{\hat{b}}\right) + \frac{\delta a}{b} y + v \right\}$$

$$= s\left\{ (\hat{a}y - a_m y_m - b_m r)\left(\frac{\hat{b} - b}{\hat{b}}\right) + \delta a\, y \right\} + bsv$$

$$\leq |s| \left\{ |\hat{a}y - a_m y_m - b_m r| \frac{\bar{b}}{\hat{b}} + \bar{a}|y| \right\} + bsv$$

Let $v = -\beta \,\mathrm{sgn}(s)$

$$\Rightarrow \quad \dot{V} \leq |s| \left\{ |\hat{a}y - a_m y_m - b_m r| \frac{\bar{b}}{\hat{b}} + \bar{a}|y| \right\} - b\beta|s|$$

$$= -b|s| \left\{ \beta - \left[\frac{|\hat{a}y - a_m y_m - b_m r| \frac{\bar{b}}{\hat{b}} + \bar{a}|y|}{b} \right] \right\}$$

$$\leq -b|s| \left\{ \beta - \left[\frac{|\hat{a}y - a_m y_m - b_m r| \frac{\bar{b}}{\hat{b}} + \bar{a}|y|}{\underline{b}} \right] \right\}$$

Let

$$\beta = \left[\frac{|\hat{a}y - a_m y_m - b_m r| \frac{\bar{b}}{\hat{b}} + \bar{a}|y|}{\underline{b}} \right] + \beta_0$$

Then

$$\dot{V} \leq -b\beta_0|s|$$

$$\leq -\underline{b}\,\beta_0\,|s|$$

$\therefore$ we can conclude finite-time convergence to the set $\{s = 0\}$

Summarizing, the control law is

$$u = -\left[\frac{\hat{a}y - a_m y_m - b_m r}{\hat{b}}\right] - \left\{\left[\frac{|\hat{a}y - a_m y_m - b_m r|\frac{\bar{b}}{\hat{b}} + \bar{a}|y|}{\underline{b}}\right] + \beta_0\right\}\operatorname{sgn}(e_1)$$

The first term is the control based on the nominal model.

The second term makes the control robust with respect to bounded uncertainty.

Matlab example

$a = -5$, $b = 2$

$\hat{a} = -3$ $\hat{b} = 1$ $\bar{b} = 3$, $\underline{b} = 0.5$, $\beta_0 = 3$

$a_m = -3$ $b_m = -3$

Sliding Mode Example 2

Simplified model of underwater vehicle heading dynamics.

$$\ddot{\psi} + a\dot{\psi}|\dot{\psi}| = \tau$$

$\psi \triangleq$ heading direction

$\tau \triangleq$ normalized torque

a - positive unknown parameter

Let $\hat{a}$ be nominal value and

$$|\delta a| = |a - \hat{a}| < \bar{a}$$

The control objective is to pick τ to follow a reference heading ψ_r

Let the sliding surface be

$$s = \dot{\tilde{\psi}} + \gamma \tilde{\psi}$$

where $\tilde{\psi} = \psi - \psi_r$ is the heading error,

and let

$$V = \tfrac{1}{2} s^2$$

Then $\dot{V} = s\dot{s}$ where

$$\dot{s} = \ddot{\tilde{\psi}} + \gamma \dot{\tilde{\psi}} = \ddot{\psi} - \ddot{\psi}_r + \gamma \dot{\tilde{\psi}} = -a\dot{\psi}|\dot{\psi}| + \tau - \ddot{\psi}_r + \gamma \dot{\tilde{\psi}}$$

Then

$$\dot{V} = s\left(-a\dot{\psi}|\dot{\psi}| + \tau - \ddot{\psi}_r + \gamma\dot{\tilde{\psi}}\right)$$

$$= s\left(-\hat{a}\dot{\psi}|\dot{\psi}| - \delta a\dot{\psi}|\dot{\psi}| + \tau - \ddot{\psi}_r + \gamma\dot{\tilde{\psi}}\right)$$

Select $\tau = \hat{a}\dot{\psi}|\dot{\psi}| + \ddot{\psi}_r - \gamma\dot{\tilde{\psi}} + v$

Then

$$\dot{V} = s\left(-\delta a\dot{\psi}|\dot{\psi}| + v\right)$$

$$\leq |s|\left(|\delta a||\dot{\psi}|^2\right) + sv$$

Select $v = -\beta\,\mathrm{sgn}(s)$

Then

$$\dot{V} \leq -|s|\left[\beta - |\delta a||\dot{\psi}|^2\right]$$

~~Select~~ ~~Because~~

$$\leq -|s|\left[\beta - \bar{a}|\dot{\psi}|^2\right]$$

Select $\beta = \bar{a}|\dot{\psi}|^2 + \beta_0$ then

$$\dot{V} \leq -\beta_0|s|$$

control is

$$\boxed{\tau = \hat{a}\dot{\psi}|\dot{\psi}| + \ddot{\psi}_r - \gamma\dot{\tilde{\psi}} - \left[\bar{a}|\dot{\psi}|^2 + \beta_0\right]\mathrm{sgn}\left(\dot{\tilde{\psi}} + \gamma\tilde{\psi}\right)}$$

< Matlab simulation >

Example 3 (Sliding Mode Observer)

Given the nonlinear plant

$$\dot{x} = Ax + f(t,x) + B[u + v(t)]$$
$$y = Cx$$

Construct a sliding mode observer to estimate the states.

Assumptions

A1) (A, C) - detectable

A2) $\exists\, Q = Q^T > 0$ and $h(t,x)$ and $w(t)$ s.t.

$$f(t,x) = P^{-1} C^T h(t,x)$$
$$B v(t) = P^{-1} C^T w(t)$$

where P satisfies

$$(A - LC)^T P + P(A - LC) = -Q$$

A3) $\exists\, \rho(y)$ s.t.

$$\| h(t,x) + w(t) \| \leq \rho(y)$$

Lets try the following observer

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x}) + S(\hat{x}, y)$$

Define $\tilde{x} = x - \hat{x}$ and let

$V = \tilde{x}^T P \tilde{x}$ where P comes from Assumption A2

then $\dot{V} = \dot{\tilde{x}}^T P \tilde{x} + \tilde{x}^T P \dot{\tilde{x}}$

where

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}}$$
$$= Ax + f(t,x) + Bu + Bv - A\hat{x} - Bu - LCx + LC\hat{x} - S(\hat{x}, y)$$

$$= (A - LC)\tilde{x} + P^{-1}C^T h(t,x) + P^{-1}C^T w(t) - S(\hat{x}, y)$$

Select $S(\hat{x}, y) = P^{-1}C^T R(\hat{x}, y)$ then

$$\dot{\tilde{x}} = (A - LC)\tilde{x} + P^{-1}C^T\left[h(t,x) + w(t) - R(\hat{x}, y)\right]$$

$\therefore$
$$\dot{V} = \tilde{x}^T(A - LC)^T P\tilde{x} + \tilde{x}^T P(A - LC)\tilde{x}$$
$$+ \left[h(t,x) + w - R(\hat{x}, y)\right]^T C P^{-T} P \tilde{x}$$
$$+ \tilde{x}^T P P^{-1} C^T \left[h(t,x) + w(t) - R(\hat{x}, y)\right]$$

$$= -\tilde{x}^T Q \tilde{x} + 2\tilde{x}^T C^T \left[h(t,x) + w(t) - R(\hat{x}, y)\right]$$

$$\Rightarrow \quad \dot{V} \le -\tilde{x}^T Q \tilde{x} + 2\|C\tilde{x}\| \|h(t,x) + w(t)\| - 2(C\tilde{x})^T R(\hat{x},y)$$

$$\le -\tilde{x}^T Q \tilde{x} + 2\|C\tilde{x}\| \rho(y) - 2(C\tilde{x})^T R(\hat{x},y)$$

Let $R(\hat{x},y) = \beta \dfrac{C\tilde{x}}{\|C\tilde{x}\|} = \beta \dfrac{(y - C\hat{x})}{\|y - C\hat{x}\|}$

Then $\dot{V} \le -\tilde{x}^T Q \tilde{x} - 2\|C\tilde{x}\| \left[\beta - \rho(y) \right]$

Let $\beta = \rho(y) + \beta_0$

$$\Rightarrow \quad \dot{V} \le -\tilde{x}^T Q \tilde{x} - 2\beta_0 \|C\tilde{x}\|$$

Therefore the sliding mode observer is

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x}) + P^{-1} C^T \left(\frac{y - C\hat{x}}{\|y - C\hat{x}\|} \right) \left[\rho(y) + \beta_0 \right]$$

<< Performance on slide smo.pdf >>

Output Feedback Control of Underwater Vehicles Using Nonlinear State Observer

Myung-Hyun Kim and Daniel J. Inman

Fourth Semi-Annual Meeting-MURI
Nonlinear Active Control of Dynamical Systems
Oct.6 1998

Outline

- Summary of Previous Work
- Problem Formulation
- Output Feedback Control
- Dive Plane Equation of Motion
- Observer Design
- Controller Design
- Simulation Results
- Conclusion

Summary of previous work

- Effects of actuator dynamics
  - Actuators in UUVs such as thrusters, rudders have time lag and this tends to degrade system performance and make quick and precise control difficult.
  - Thruster dynamics has significant influence on the dynamics of vehicle.
  - Investigation of the influence of the rudder dynamics on the system response. However, since UUVs are operated in low speed, rudder dynamics does not influence the UUV dynamics.
  - Rudder dynamics is substantial only at high speed such as in torpedo application.

Problem Formulation

- Sliding Mode Controller
  - Advantages
    - Capable of handling nonlinear systems
    - Robust to uncertainties/parameter variation
  - Disadvantages
    - All states measurement required
    - Occurrence of undesirable chattering phenomena
  - Chattering can be avoided by including boundary layer
  - Design of nonlinear observer for state estimation

Output Feedback Control

- Healey(NPS) et.al.
  - Design of Robust Observer
  - Sliding Mode Control with estimated states
- Fossen(NIT) et.al.
  - Nonlinear Observer from the kinematic relationship with additional acceleration measurements
  - Adaptive Control with estimated states
- Nonlinear State observer using Sliding Mode

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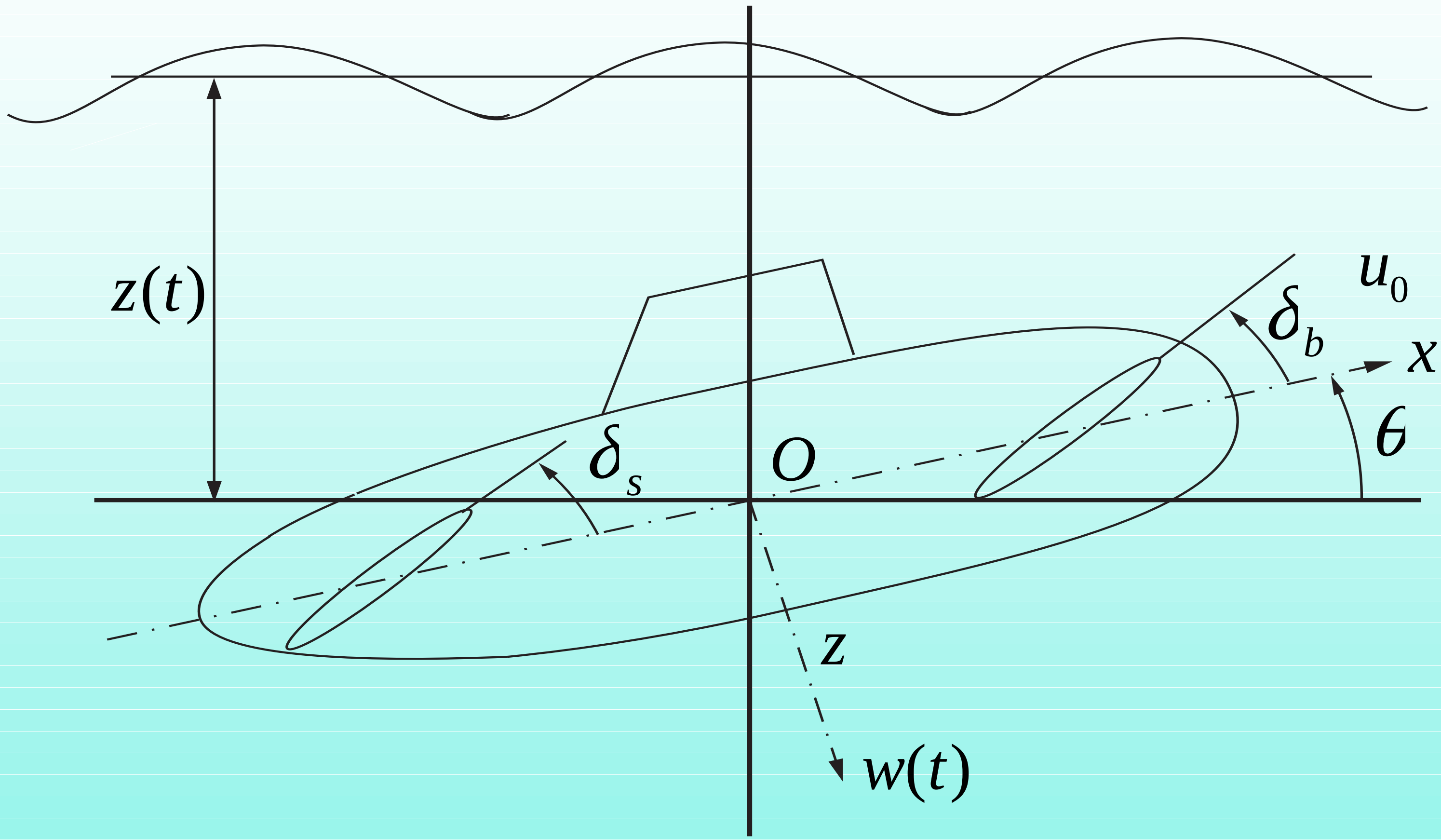
Dive Plane Equation of Motion



Dive Plane Equation of Motion(Cont'd)

$$m[\dot{w} - uq - x_G \dot{q} - z_G q^2]$$
$$= z_{\dot{q}} \dot{q} + z_{\dot{w}} \dot{w} + z_q uq + z_w w + u^2 (z_b \delta_b + z_s \delta_s)$$

$$I_y \dot{q} - m[x_G(\dot{w} - uq) - z_G(\dot{u} + wq)$$
$$= M_{\dot{q}} \dot{q} + M_{\dot{w}} \dot{w} + M_q uq + M_w uw + u^2 (M_b \delta_b + M_s \delta_s)$$
$$-(x_G mg - x_B B)\cos\theta - (z_G mg - z_B B)\sin\theta$$

$$\dot{\theta} = q$$

$$\dot{z} = w\cos\theta - u\sin\theta$$

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Dive Plane Equation of Motion(Cont'd)

State Space Form

$$\begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{\theta} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} a_{11}u & a_{12}u & a_{13} & 0 \\ a_{21}u & a_{22}u & a_{23} & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -u & 0 \end{bmatrix} \begin{bmatrix} w \\ q \\ \theta \\ z \end{bmatrix} + \begin{bmatrix} b_{11}u^2 & b_{12}u^2 \\ b_{21}u^2 & b_{22}u^2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_b \\ \delta_s \end{bmatrix}$$

$$+ \begin{bmatrix} F_d \cos\theta \\ M_d \cos\theta \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} e_{11}q^2 + e_{12}qw \\ e_{21}q^2 + e_{22}qw \\ 0 \\ 0 \end{bmatrix}$$

Output Equation

$$\begin{bmatrix} z \\ q + \theta \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ q \\ \theta \\ z \end{bmatrix}$$

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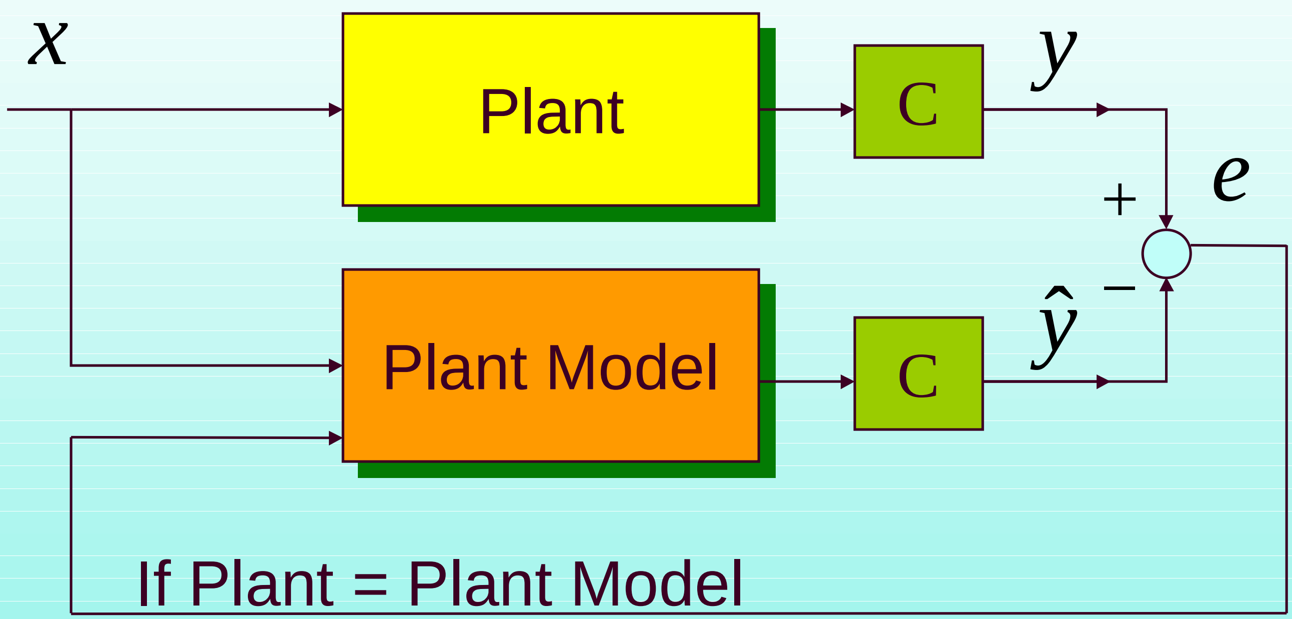
Observer Design



If Plant = Plant Model

then $y = \hat{y}$

Observer Design(Cont'd)

$$\dot{x} = Ax + f(t,x) + B[u(t) + v(t)]$$

$$y = Cx$$

Assumptions

A1) The system is detectable so that we can find G.
The spectrum of $A - GC = 0$ is in the LHP

A2) There exists $Q \in R^{n \times m}$,symmetric and positive definite,
and function h and w such that:

$$f(t,x) = P^{-1}C^T h(t,x)$$

$$Bv(t) = P^{-1}C^T w(t)$$

where $A_0^T P + PA_0 = -Q$

A3) Let $\xi(t,x) = h(t,x) + w(t,x)$. Then, there exists a positive scalar
Such that: $\|\xi(t,x)\| < \rho$

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Observer Design(Cont'd)

Sliding Mode Observer

$$\dot{\hat{x}} = A_o \hat{x} + Gy + S(\hat{x}, y) + Bu$$

where
$$S(\hat{x}, y) = \begin{cases} -\dfrac{P^{-1}C^T Ce}{\|Ce\|}\rho & \text{for } e \neq 0 \\ 0 & \text{for } e = 0 \end{cases}$$

- We measure the depth and the pitch rate.
- The pitch angle can be obtained by integrating pitch rate.

Controller Design

- PD Controller

$$\begin{bmatrix} \delta_b \\ \delta_s \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \end{bmatrix} \begin{bmatrix} w - w_{com} \\ q - q_{com} \\ \theta - \theta_{com} \\ z - z_{com} \end{bmatrix}$$

- Sliding Mode Controller

Switching surface

$$\sigma(t,x) = [\sigma_1(t,x) \quad \ldots \quad \sigma_m(t,x)]^T = 0$$

Reaching condition

$$\frac{d}{dt}\left(\frac{1}{2}\sigma^T \sigma\right) = \sigma^T \dot{\sigma} < 0$$

Controller Design(Cont'd)

Gradient of σ

$$\frac{\partial \sigma(t,x)}{\partial x} = S(t,x)$$

Control input

$$u = u_{eq_{nom}} + \bar{u}$$

$$\begin{cases} u_{eq_{nom}} = -[S(t,x)B]^{-1}(S(t,x)Ax + \frac{\partial \sigma}{\partial t}) \\ \bar{u} = -[S(t,x)B]^{-1}\eta\, sign(\sigma) \end{cases}$$

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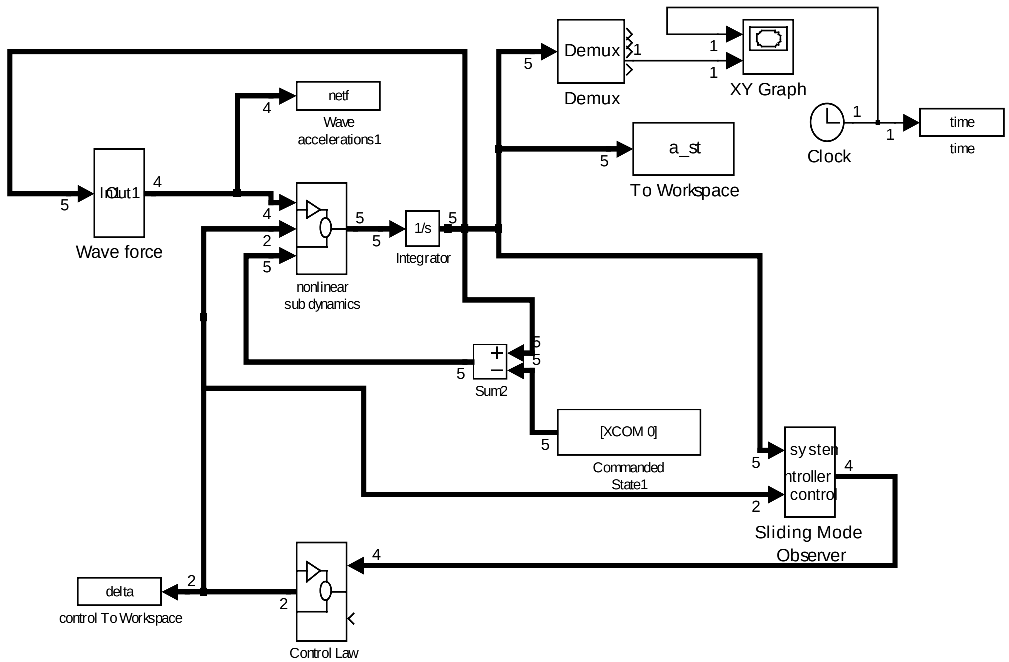
Simulation results

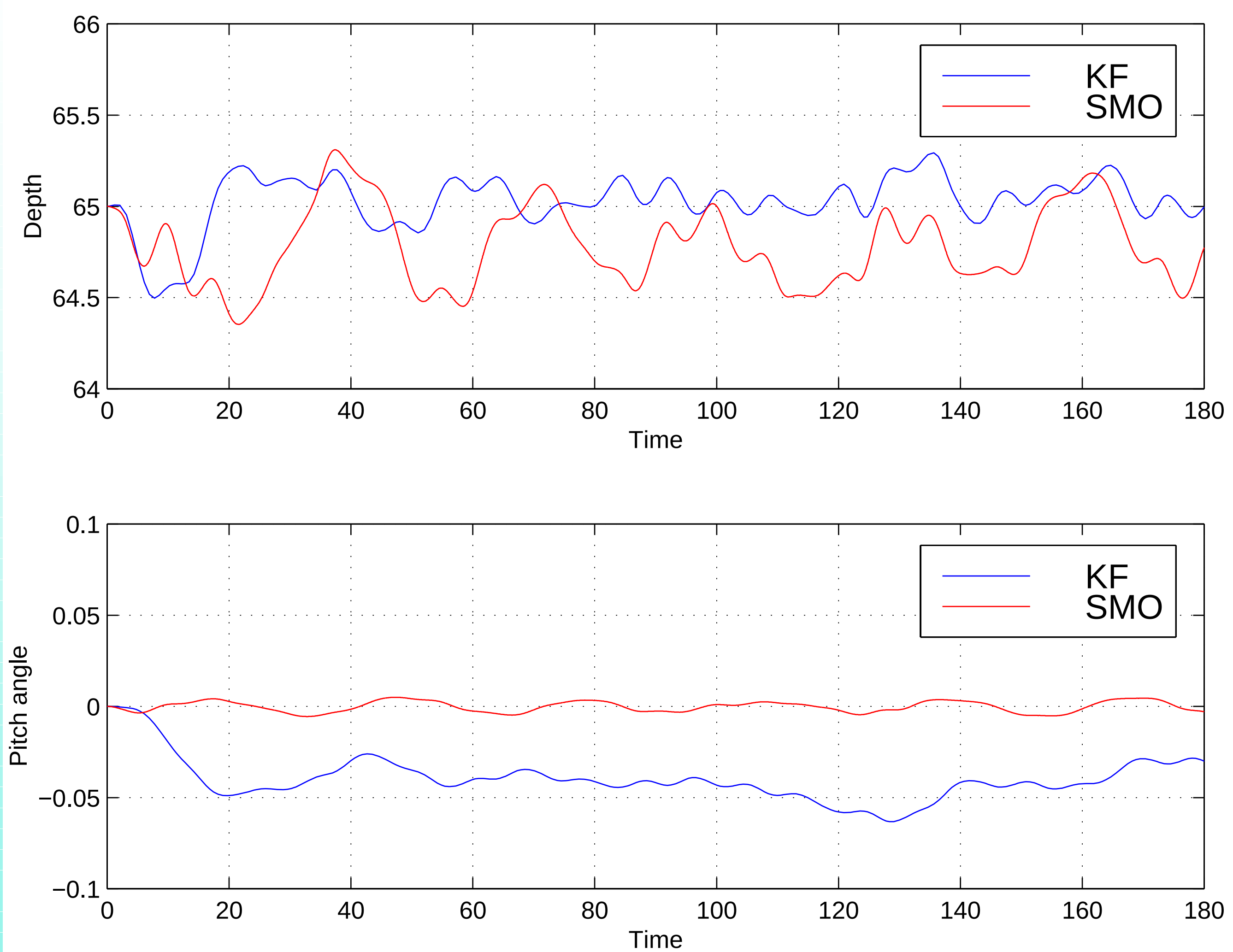
- Nominal forward speed 6 knot
- Bounds on nonlinear terms

$\lvert\theta\rvert < 10^o$	: pitch angle
$\lvert q\rvert < 0.2 \quad rad/\sec$	: pitch rate
$\lvert w\rvert < 1.5 \quad ft/s$	: heave speed

- Include bounds of wave force

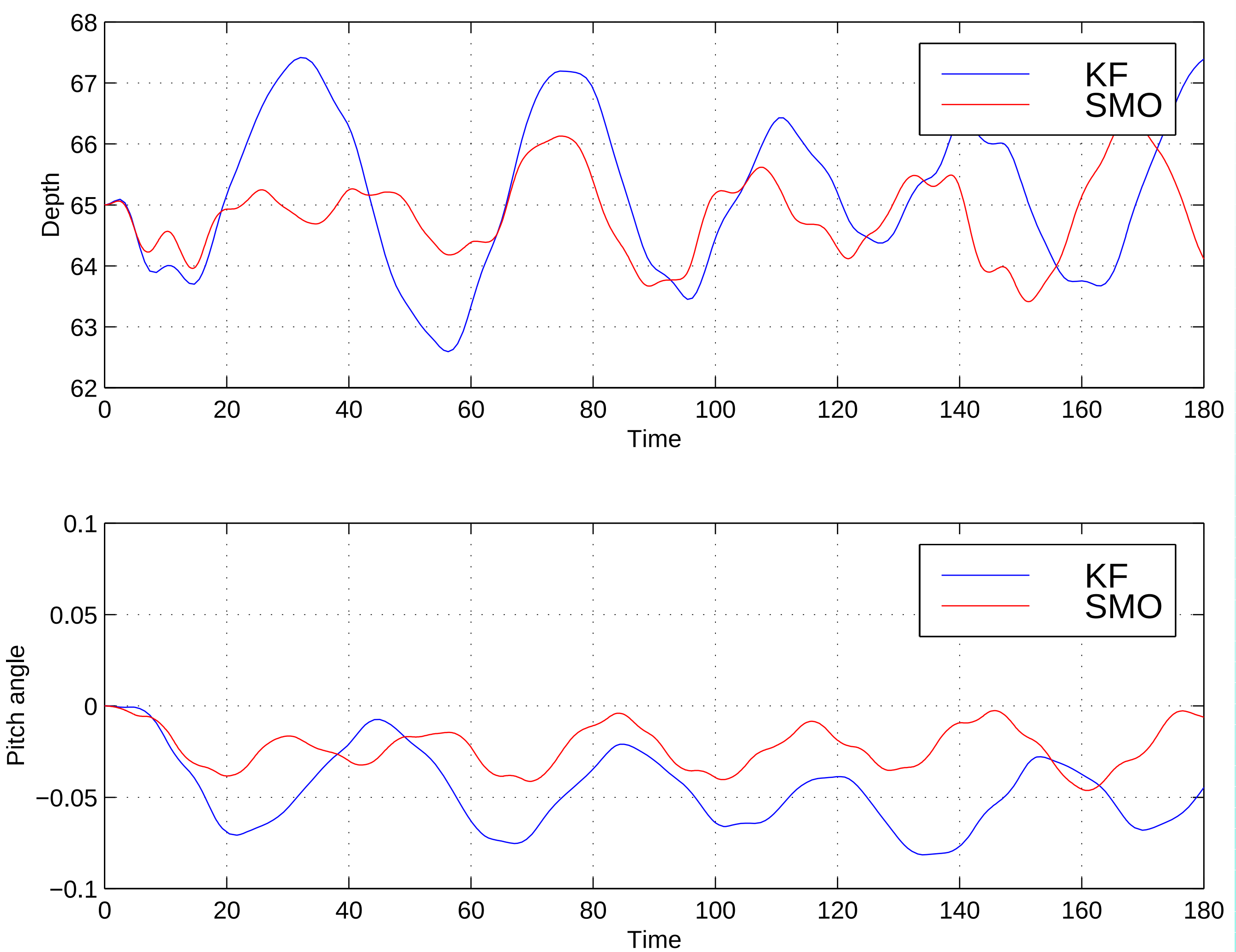
Simulink Model



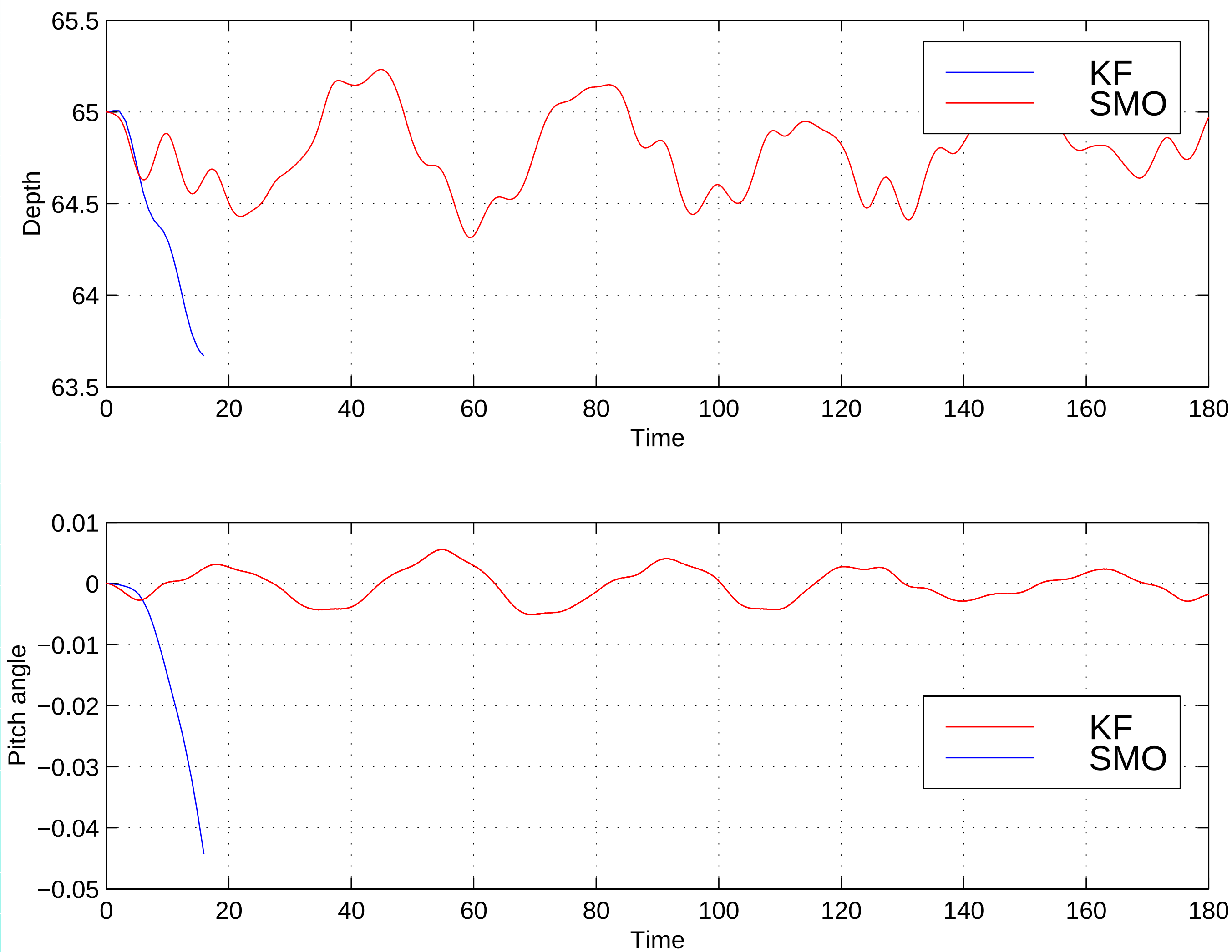


Depth and pitch response to 65ft dive command at sea state 3

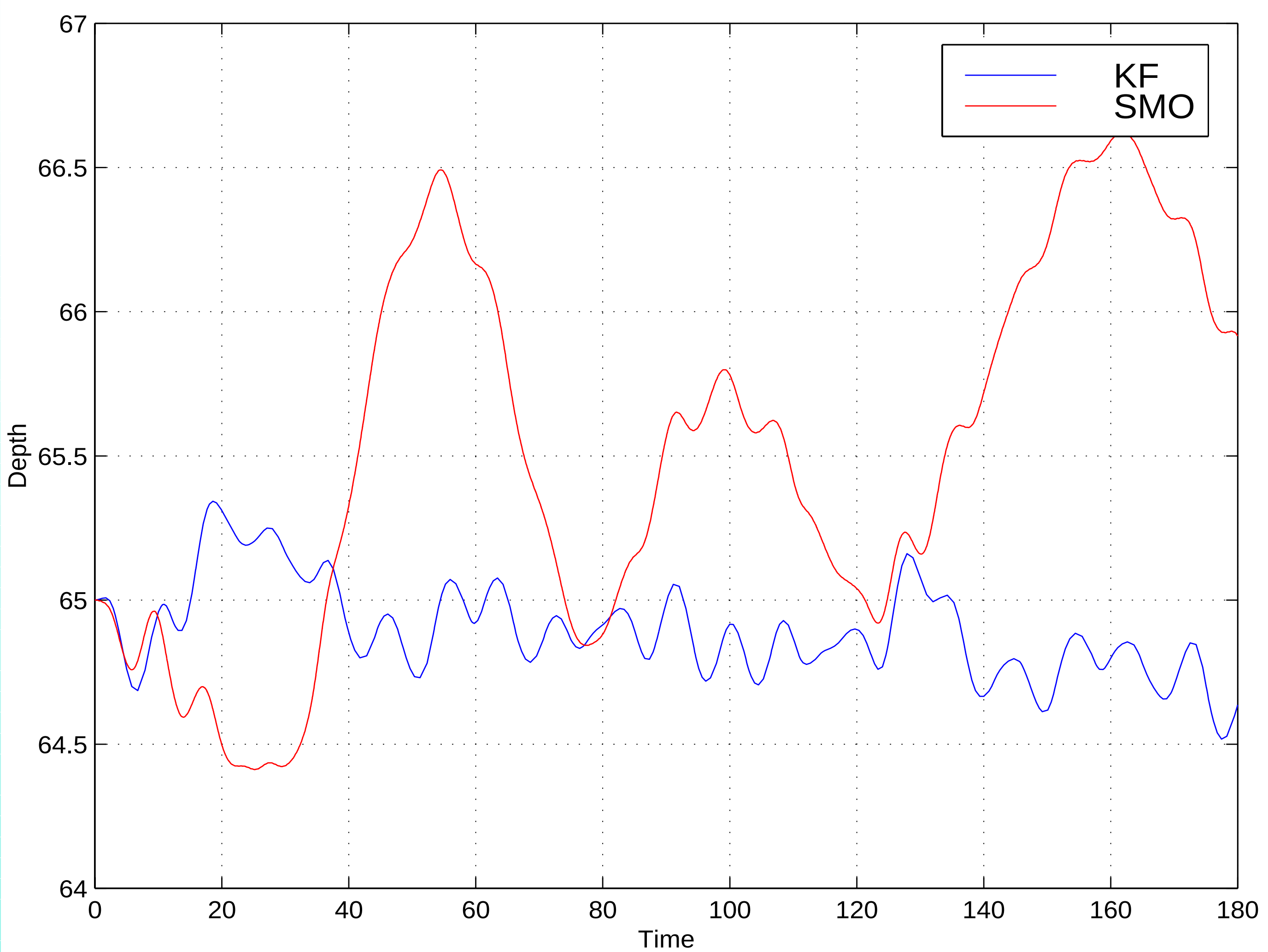
-PD control-



Depth and pitch response to 65ft dive command at sea state 4
-PD control-

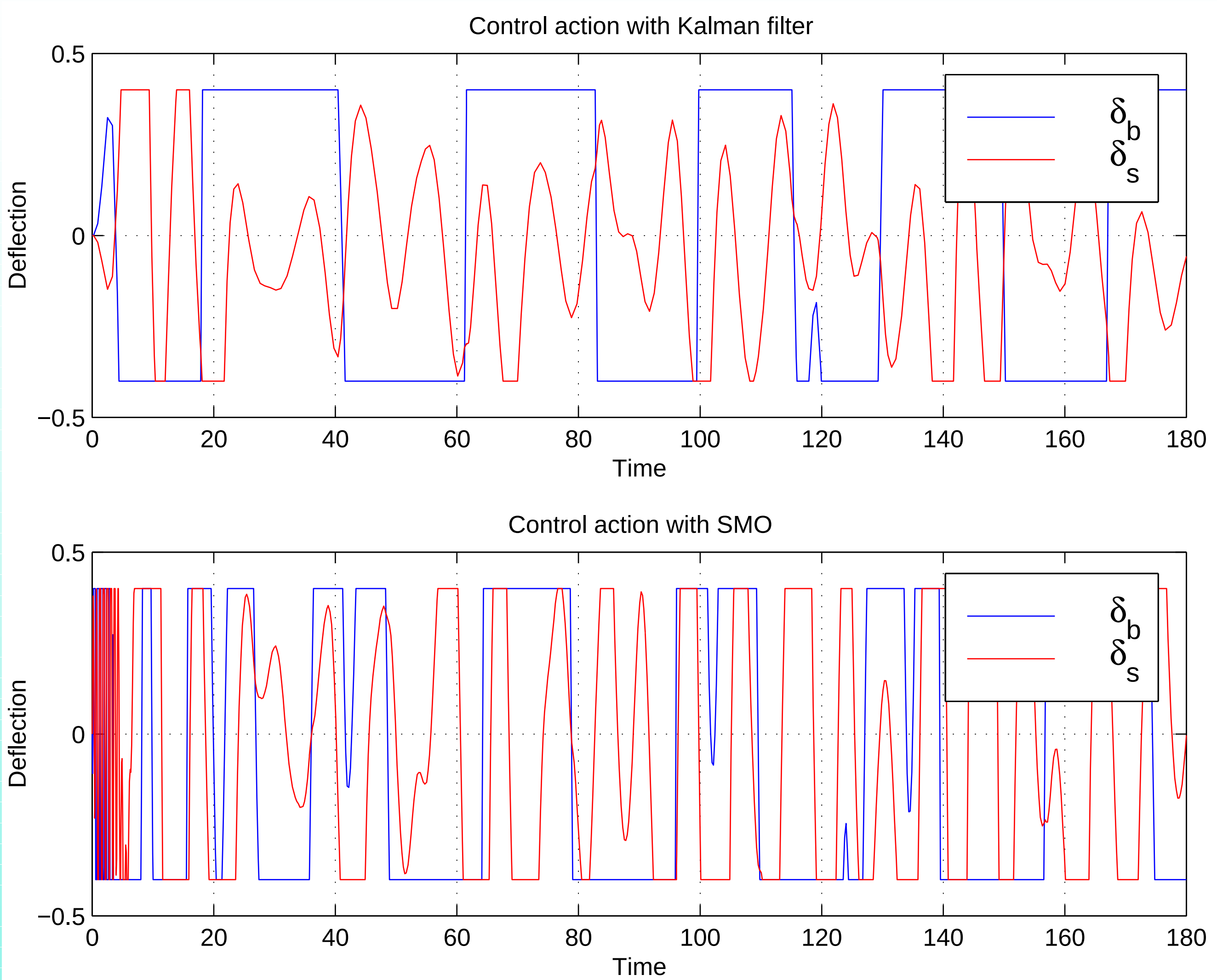


Depth and pitch response to 65ft dive command
with 30% parameter variation
-PD control-



Depth and pitch response to 65ft dive command at sea state 3

-Sliding mode control-

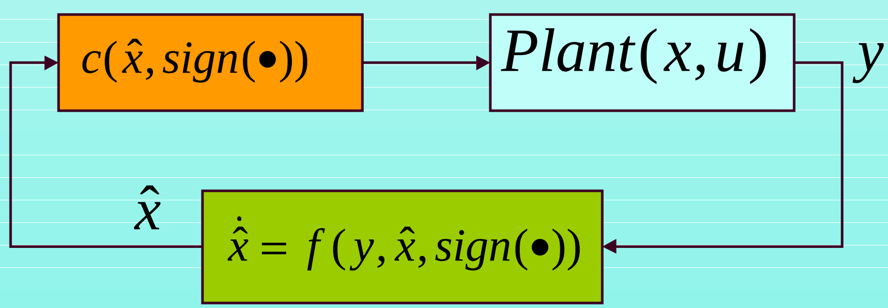


Comparison of control action

SMC and SMO Combination

- Unexpected results are observed when SMC(Sliding Mode Controller) and SMO(Sliding Mode Observer) are combined.
- Possible problem
  - nested discontinuities

$$f = sign(sign(u))$$



Conclusion

- Sliding mode observer is designed and applied in the dive plane of submarine in order to estimate linear velocity component(heave velocity).
- It is compared with the Kalman filter estimator which is based on linearized dynamics and showed better performance in terms of robustness to wave disturbance, model parameter uncertainties, and parameter variation.
- When,however, the SMC and SMO are combined, undesirable results are obtained. More investigation is required.
- This sliding mode observer can be applied to fault detection algorithm in replace of Kalman filter.
- It takes more computational time in estimating the states when sliding mode observer is used. More efficient computational algorithm needs to be investigated.