Lab: Kalman Filter for Three Wheeled Robot

ECEn 483

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1 Introduction

The objective of this lab is to design an observer for the three wheeled robot based on Kalman filter theory. The Kalman filter will be used to

1. Filter noise from the vision system,
2. Estimate the current position and angle of the robot,
3. Estimate position and angle between measurements.

2 Mathematical Model

Earlier in the semester we developed a velocity controller for the three wheeled robots. We will assume that the velocity controller have been implemented but that they are not perfect. In other words, the actual velocity of the robot is given by

$$\begin{aligned} V_x &= V_x^c + \delta V_x \\ V_y &= V_y^c + \delta V_y \\ \omega &= \omega^c + \delta\omega, \end{aligned}$$

where V_x^c, V_y^c, and ω^c are commanded velocities and δV_x, δV_y, and $\delta\omega$ are deviations. We will assume that the deviations are zero mean Gaussian random variables with covariance Q. Therefore, a kinematic model of the robot is given by

$$\begin{aligned} \dot{r}_x &= V_x = V_x^c + \delta V_x \\ \dot{r}_y &= V_y = V_y^c + \delta V_y \\ \dot{\theta} &= \omega = \omega^c + \delta\omega. \end{aligned}$$

Using the cameras to provide a measurement of the position and the angle, we get

$$\begin{aligned} y_{r_x} &= r_x + \eta_1 \\ y_{r_y} &= r_y + \eta_2 \\ y_\theta &= \theta + \eta_3, \end{aligned}$$

where $\eta = (\eta_1, \eta_2, \eta_3)^T$ is a zero mean Gaussian random variable with covariance R.

Therefore, the state space model is given by

$$\begin{pmatrix} \dot{r}_x \\ \dot{r}_y \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} r_x \\ r_y \\ \theta \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_x^c \\ V_y^c \\ \omega^c \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \delta V_x \\ \delta V_y \\ \delta\omega \end{pmatrix}$$

$$\begin{pmatrix} y_{r_x} \\ y_{r_y} \\ y_\omega \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_x \\ r_y \\ \theta \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix},$$

where $A = 0$, $B = I$, $G = I$, and $C = I$.

3 Kalman Filter

The continuous-discrete Kalman filter is given by the following algorithm.

System Model.

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + Gw \\ z(kT) &= C\hat{x}(kT) + v_k, \end{aligned}$$

where $w \sim \mathcal{N}(0, Q)$ and $v_k \sim \mathcal{N}(0, R)$.

Initialization. The first step is to initialize $\hat{x}$ and P to $\hat{x}_0$ and P_0.

Time update. In between measurements, the state estimate $\hat{x}$ and the covariance matrix P are propagated according to the differential equations

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu \\ \dot{P} &= AP + PA^T + GQG^T. \end{aligned}$$

In the lab, these differential equations will need to solved numerically. This can be done by approximating the time derivative with an Euler approximation, and propagating the following difference equations

$$\hat{x}[k+1] = \hat{x}[k] + T_p \left(A\hat{x} + Bu\right) \quad (1)$$

$$P[k+1] = P[k] + T_p \left(AP[k] + P[k]A + GQG^T\right), \quad (2)$$

where T_p is the sample rate of propagation, $T_p << T_s$, and T_s is the sample rate of the measurement.

Measurement update. When a measurement is received, we stop the time propagation and let $\hat{x}^-$ and P^- be the values of $\hat{x}$ as currently computed by Equations (1) and (2), respectively. The values of $\hat{x}$ and P are revised, according to the measurement, as follows:

$$L = P^- C^T [C P^- C^T + R]^{-1} \tag{3}$$

$$P = (I - LC)P^- \tag{4}$$

$$\hat{x} = \hat{x}^- + L(z_k - C\hat{x}^-). \tag{5}$$

4 Application to Three Wheeled Robots

The Kalman filter equations are particularly simple for three wheeled robots. In particular, notice that if Q and R are diagonal matrices, then P and L are always diagonal matrices. Making this assumption we get the following algorithm.

Initialization. Set $\hat{x} = (r_x(0), r_y(0), \theta(0))^T$ as estimated by averaging several camera samples. Set $P = diag([p_1, p_2, p_3]) = diag([1, 1, 1])$, or $p_1 = p_2 = p_3 = 1$.

Time update. In between measurements propagate the following differential equations:

$$\begin{aligned}
\dot{\hat{r}}_x &= V_x^c \\
\dot{\hat{r}}_y &= V_y^c \\
\dot{\hat{\theta}} &= \omega^c \\
\dot{p}_1 &= q_1 \\
\dot{p}_2 &= q_2 \\
\dot{p}_3 &= q_3.
\end{aligned}$$

The Euler approximation of these equations is given by

$$\begin{aligned}
\hat{r}_x[k+1] &= \hat{r}_x[k] + T_p V_x^c \\
\hat{r}_y[k+1] &= \hat{r}_y[k] + T_p V_y^c \\
\hat{\theta}[k+1] &= \hat{\theta}[k] + T_p \omega^c \\
p_1[k+1] &= p_1[k] + T_p q_1 \\
p_2[k+1] &= p_2[k] + T_p q_2 \\
p_3[k+1] &= p_3[k] + T_p q_3.
\end{aligned}$$

Measurement update. At the measurement we have

$$
\begin{aligned}
l_1 &= \frac{p_1^-}{p_1^- + r_1} \\
l_2 &= \frac{p_2^-}{p_2^- + r_2} \\
l_3 &= \frac{p_3^-}{p_3^- + r_3} \\
p_1 &= (1 - l_1)p_1^- \\
p_2 &= (1 - l_2)p_2^- \\
p_3 &= (1 - l_3)p_3^- \\
\hat{r}_x &= \hat{r}_x^- + l_1(y_{p_x} - \hat{r}_x^-) \\
\hat{r}_y &= \hat{r}_y^- + l_2(y_{p_y} - \hat{r}_y^-) \\
\hat{\theta} &= \hat{\theta}^- + l_3(y_\theta - \hat{\theta}^-).
\end{aligned}
$$

5 Requirements

Implement a Kalman filter for a three wheeled robot in Simulink.