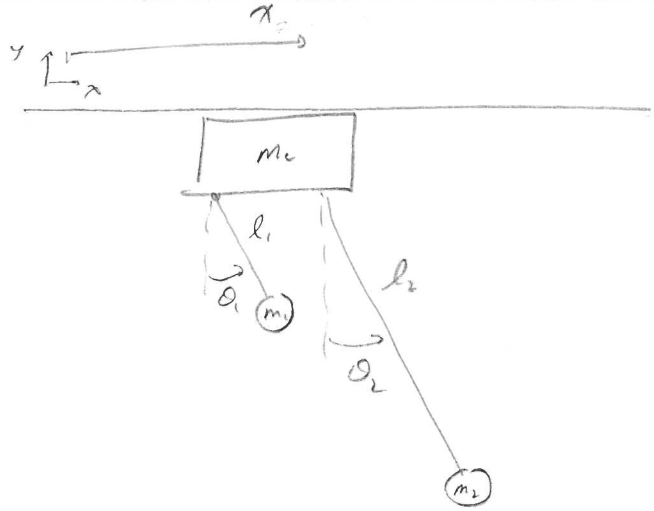
Solution

VI.1



position of cart $\quad p_c = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}$

position of m_1 $\quad p_1 = \begin{pmatrix} x + p_{offset} + l_1 \sin\theta_1 \\ offset - l_1 \cos\theta_1 \\ 0 \end{pmatrix}$

position of m_2 $\quad p_2 = \begin{pmatrix} x + offset + l_2 \sin\theta_2 \\ offset - l_2 \cos\theta_2 \\ 0 \end{pmatrix}$

$\therefore$ velocities are

$$v_c = \begin{pmatrix} \dot{x} \\ 0 \\ 0 \end{pmatrix}, \quad v_1 = \begin{pmatrix} \dot{x} + \dot{\theta}_1 l_1 \cos\theta_1 \\ \dot{\theta}_1 l_1 \sin\theta_1 \\ 0 \end{pmatrix}, \quad v_2 = \begin{pmatrix} \dot{x} + \dot{\theta}_2 l_2 \cos\theta_2 \\ \dot{\theta}_2 l_2 \sin\theta_2 \\ 0 \end{pmatrix}$$

$\therefore$ The kinetic energy is

$$K = \tfrac{1}{2} m_c v_c^T v_c + \tfrac{1}{2} m_1 v_1^T v_1 + \tfrac{1}{2} m_2 v_2^T v_2$$

$$= \tfrac{1}{2} m_c \dot{x}^2 + \tfrac{1}{2} m_1 \left[(\dot{x} + \dot{\theta}_1 l_1 \cos\theta_1)^2 + (\dot{\theta}_1 l_1 \sin\theta_1)^2 \right]$$
$$+ \tfrac{1}{2} m_2 \left[(\dot{x}_1 + \dot{\theta}_2 l_2 \cos\theta_2)^2 + (\dot{\theta}_2 l_2 \sin\theta_2)^2 \right]$$

$$= \tfrac{1}{2} \underline{m_c} \dot{x}^2 + \tfrac{1}{2} m_1 \left[\dot{x}^2 + 2\dot{\theta}_1 \dot{x} \underline{l_1 \cos\theta_1} + \dot{\theta}_1^2 l_1^2 \cos^2\theta_1 + \dot{\theta}_1^2 l_1^2 \sin^2\theta_1 \right]$$
$$+ \tfrac{1}{2} m_2 \left[\dot{x}^2 + 2\dot{\theta}_2 \dot{x} \underline{l_2 \cos\theta_2} + \dot{\theta}_2^2 l_2^2 \cos^2\theta_2 + \dot{\theta}_2^2 l_2^2 \sin^2\theta_2 \right.$$

$$\therefore \; K = \tfrac{1}{2}(m_c + m_1 + m_2)\dot{x}^2 + \tfrac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \tfrac{1}{2} m_2 l_2^2 \dot{\theta}_2^2$$
$$+ m_1 l_1 \dot{\theta}_1 \dot{x} \cos\theta_1 + m_2 l_2 \dot{\theta}_2 \dot{x} \cos\theta_2$$